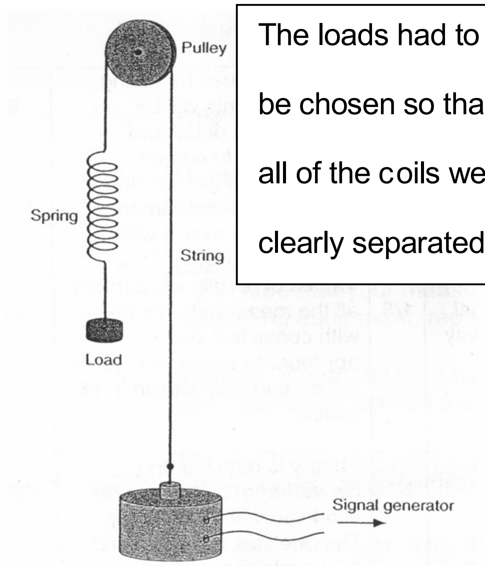
An Experiment to Investigate a Mass on a spring as an Example of Resonance

Method

We set up the apparatus as shown below. We also included a meter rule to the left of the spring so that we could see the size of the oscillations.



The loads had to be chosen so that all of the coils were clearly separated.

We set the signal generator to produce a sine wave output and set both the frequency and amplitude to a minimum. We switched on the signal generator and set the amplitude to its middle setting. We pulled down gently on the load and allowed the spring to oscillate. We slowly increased the frequency, monitoring the amplitude of the oscillations of the load by reading from the meter rule placed next to the apparatus. We noted when the amplitude appeared to be at its largest and took this frequen cy to be the resonant frequency. We repeated the experiment using different masses and decided to repeat each experiment 3 times for comparison.

Measurements

Before commencing the experiment, we considered what precautions we could take to ensure accuracy. We placed a meter rule by the apparatus to give us the best possible chance of observing the largest amplitude correctly. We weighed the entire spring system (Weights, hanger and spring as all of these items were involved in the actual oscillation that we were measuring) each time we changed the mass of the system to ensure and accurate reading for mass. We felt that just adding weights and assuming that the system would increase by exactly 0.1kg would leave too much room for error. We recorded all of our measurements which are shown below.

We felt that we could improve our accuracy further by performing a couple of trial experiments to see how we could improve upon our initial ideas. Our trials did not suggest any obvious improvements so we continued with the above method and recorded the following results.

Mass (Kg)	1/M (1/kg)	Resonant Frequency 1 (Hz)	Resonant Frequency 2 (Hz)	Resonant Frequency 3 (Hz)	Average Resonant Frequency (f) (Hz)	f^2 (Hz^2)
0.107	9.35	2.4	2.3	2.4	2.37	5.62
0.205	4.88	1.6	1.7	1.7	1.67	2.79
0.303	3.30	1.5	1.4	1.4	1.43	2.04
0.400	2.50	1.2	1.3	1.3	1.27	1.61
0.499	2.00	1.2	1.2	1.2	1.20	1.44

We noted that, in general, that frequency decreased with mass.

Theory

The theory is that resonance occurs at the point where the natural frequency of the spring system is equal to the frequency of the signal generator.

We know that the time period for a mass on a spring is given by

$T= (2\pi)(\sqrt{(m/k)})$ but we also know that $f = 1/T$ so

$f = 1/T = 1/ ((2\pi) (\sqrt{(m/k)})$

$= (1/(2\pi)) (1/(\sqrt{(m/k)})$

$= (1/(2\pi)) (\sqrt{(k/m)}$

So

$f^2 = ((1/(2\pi))\,(\sqrt{(k/m)})^2$

$= (1/(2\pi)^2\,(\sqrt{(k/m)})^2$

$= (1/4\pi^2)(k/m)$

$= (k/4\pi^2)(1/m)$

I have included columns in the results table for 1 /m and f^2 as $f^2= (k/4\pi^2)(1/m)$ which is in the form y=mx+c. This means that a graph of f^2 plotted against 1/m should give us a straight line with a gradient of $k/4\pi^2$, which means we will be able to find the spring constant k (See graph 1).

From my graph the gradient = 0.568 so

$0.568 = k/4\pi^2$ which means

$k = 0.568 \times (4\pi^2) = 22.42$ N/m

In order to verify this we performed a further experiment. Using the equipment set up in its original format, we taped the string to the meter rule in order to keep the spring stationery. We then measured the extensions of the spring, at rest, firstly without weights and then with the individual weights previously used. We recorded the following results.

Mass (Kg)	Spring extension (m)	Spring Constant K (kg/m)	Spring Constant k (N/m)
0.107	0.041	2.61	25.6
0.205	0.081	2.53	24.82
0.303	0.120	2.53	24.82
0.400	0.161	2.48	24.33
0.499	0.202	2.47	24.23

We acknowledged that k should be the same for each mass and noted that k was similar for each result, being approximately 24 .76 N/m.

However, we need to consider the following errors.

Errors - first experiment

Source of error	Average	½ smallest division	½ range	Use	%
Signal generator (0.2 hz smallest division)	1.43	0.1 hz	(1.5-1.4)/2 = 0.05Hz	0.1Hz	6.99
Scales (0.01 smallest division)	3.30	0.005g	Not Applicable	0.005	0.15
Total % error (Total *)					7.14

Note that the meter rule has not been included in the above error table as we did not use it to take measurements and no measurements of length from the meter rule were used in any calculations based on the results of this experiment.

After considering the above table of errors, we amend the natural resonance of the spring recorded from the first experiment as follows: -

k = 22.42 N/m +/- 7.14% = 22.42 +/- 1.60 N/m

This would mean that the top end of our range is k = 24.02 N/m. The second experiment gave us k = 24.76 N/m. However, we need to take into account the error of the meter rule.

Errors – second experiment

Source of error	Average	½ smallest division	½ range	Use	%
Meter rule (0.001m smallest division)	0.12m	0.0005m	Not applicable	0.0005	0.42

So, according to our second experiment, k = 24.76N/m +/ - 0.42% = 0.10N/m

This means that the bottom end of our range is 24.66N/m.

So the smallest difference between the value for k from the first experiment and the value for k from our second experiment is 0.64N/m. Whilst this means that the value of the second experiment is out of range, the difference between the first 2 figures is only 2.66% of our top end value for k from the first experiment.

It therefore seems fair to say that our graph appears to support the hypothesis that resonance occurs when the driving frequency is equal to the natural frequency. I feel that further readings would improve the graph and perhaps yield a more accurate value for k.

In order to improve the experiment I would attempt to measure the natural resonance of the spring using a stop watch and the meter rule for comparison purposes. We could also add a Perspex tube in which to place the spring and load to prevent the spring from swinging. However, we would need to ensure that the spring did not hit the side as this may affect results.

Ideas for further research

We could research whether or not a spring moving i n any direction other that up and down, ie swinging during the experiment would materially change the results. We could also investigate what would happen if we damped the oscillation by repeating the experiment with the load suspended in water. Initial thoughts would be that the velocity of the oscillations may be reduced but we would be more concerned with whether or not the amplitude of the wave had changed and thus the frequency of the natural resonance,