

Investigating the relationship of projectile range and projectile motion using a ski jump.Introduction

As we all know ski jumping is a worldwide sport in which athletes skate down a slope ramp, gaining speed that throws them in the air that makes them land some distance away. The distance travelled at the time when the jumper leaves the ramp, until he reaches the ground is known as the jump range. This interesting and challenging sport involves a lot of physics behind it. Kinetic energy, gravitational potential energy, motion, speed, height, time, distance and the athlete's ability to reduce air resistance to their body are all factors that determine the athlete's performance. This experiment represents a ski jumping slope through which we will investigate and demonstrate how physics can be used by ski jumpers to increase their range in the jump.

Aim: My aim of this experiment is to explore the relationship between the launch height and the range of the jump. I will use my knowledge of physics knowing that gravitational potential energy can be converted into kinetic energy and using the equations $\Delta E_{grav} = mg\Delta h$ for gravitational potential energy and $E_k = \frac{1}{2}mv^2$ for the kinetic energy to work out the relationship between height, velocity and the range of the projectile.

Hypothesis

I predict that the higher the ramp, the higher the velocity therefore an increase in range.

Null Hypothesis

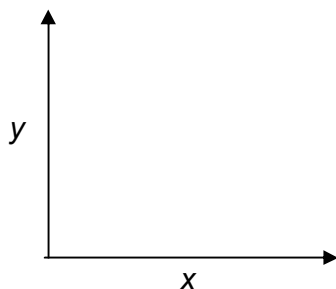
There will be no relationship found between the height of the ramp, velocity and the range of the projectile.

Rationale

The displacement, velocity and acceleration of the projectile are all vectors. The forces of the projectile motion can be treated separately, so they can be resolved into horizontal and vertical components. They are independent of each other; that is, neither motion affects the other. We can label these components x and y . The horizontal component can be represented as V_x and the vertical component as V_y .

$$V_x = V \cos \theta$$

$$V_y = V \sin \theta$$



The projectile/ball bearing will be dropped from the top of the ramp giving the projectile gravitational potential energy. We can work out the gravitational potential energy by using $\Delta E_{grav} = mg\Delta h$ as we know the mass of the projectile, gravitational effect and the height at which the projectile is launched. As the projectile travels down the ramp it will be affected by air resistance and friction, however in this experiment we will ignore these factors. The point at which the projectile starts moving down the ramp, the gravitational potential energy converts into kinetic energy allowing it to accelerate off the ramp and into the air. We can use the equation $E_k = \frac{1}{2}mv^2$ to work out the kinetic energy. Ignoring air resistance and friction, we know that the gravitational potential energy at the top of the ramp will equal the kinetic energy when it leaves at the bottom. This experiment involves me looking at the relationship between the height and the range of the projectile. In theory by changing the height I am changing the velocity of the projectile and so therefore the range will be affected.

By re-arranging the equation $E_k = \frac{1}{2}mv^2$ we can work out the velocity of the ball bearing, $2E_k/m = v^2$ then do $\sqrt{v^2} = v$. This can also be simplified to give $\sqrt{(2mgh)/m} = v$. The "m" cancels which will leave us with $V = \sqrt{(2gh)}$. We can then use the equation $V = \text{Distance}/\text{Time}$ which will give the theoretical horizontal value. We can re-arrange this equation to give us the distance, $\text{Distance} = \text{Velocity} \times \text{Time}$. This is the horizontal component.

Changing the vertical height of the drop from the ramp would vary the range. This is because according to the law of energy, it cannot be made or destroyed but transferred. To apply this law into this practical, we can say all of the gravitational potential energy is converted into kinetic energy assuming no energy is wasted. As a result, gravitational potential energy is equal to kinetic energy.

I will also need to calculate the time by using the equation

$$S = u + \frac{1}{2}at^2$$

s = the distance from floor,
t = time
u = 0 (Vertical velocity) and
a = gravity = 9.81

We can simplify this equation to $s = \frac{1}{2}at^2$

Or

$$2s/g = t^2$$

Where we can change it to give me

$$t = \sqrt{(2s / g)}$$

This is the vertical component.

Safety:

When it comes to a practical experiment it is always important to consider health and safety. In this experiment, we are using a ball bearing and so if due care isn't taken then this could result in minor injuries especially to those who are carrying out the investigation or walking past. I need to make sure the landing zone is clear of obstacles. I will make sure there are no breakable objects around because the metal ball will bounce for a distance before it stops.

Variables

Dependent: The distance between the table and the point of contact of the ball bearing.

Independent: Height/the point at which the ball bearing is dropped from.

Control:

- The point at which the ball bearing is dropped.
- The angle of the jump.
- Mass of the ball bearing

For this experiment I aim to change the height of the ramp by 10 centimetres each time and test each height 4 times. I will use a meter ruler stating centimetres and millimetres to measure the height of the ramp from the table to the point of launch and measure the range between the table and the point at which the ball bearing contacted the ground. It is important for me to bear in mind that my results may vary due to the forces of air resistance and friction throughout the experiment. To increase the accuracy of my results, I will make sure that the dropping point on the ramp will be the same throughout my investigation.

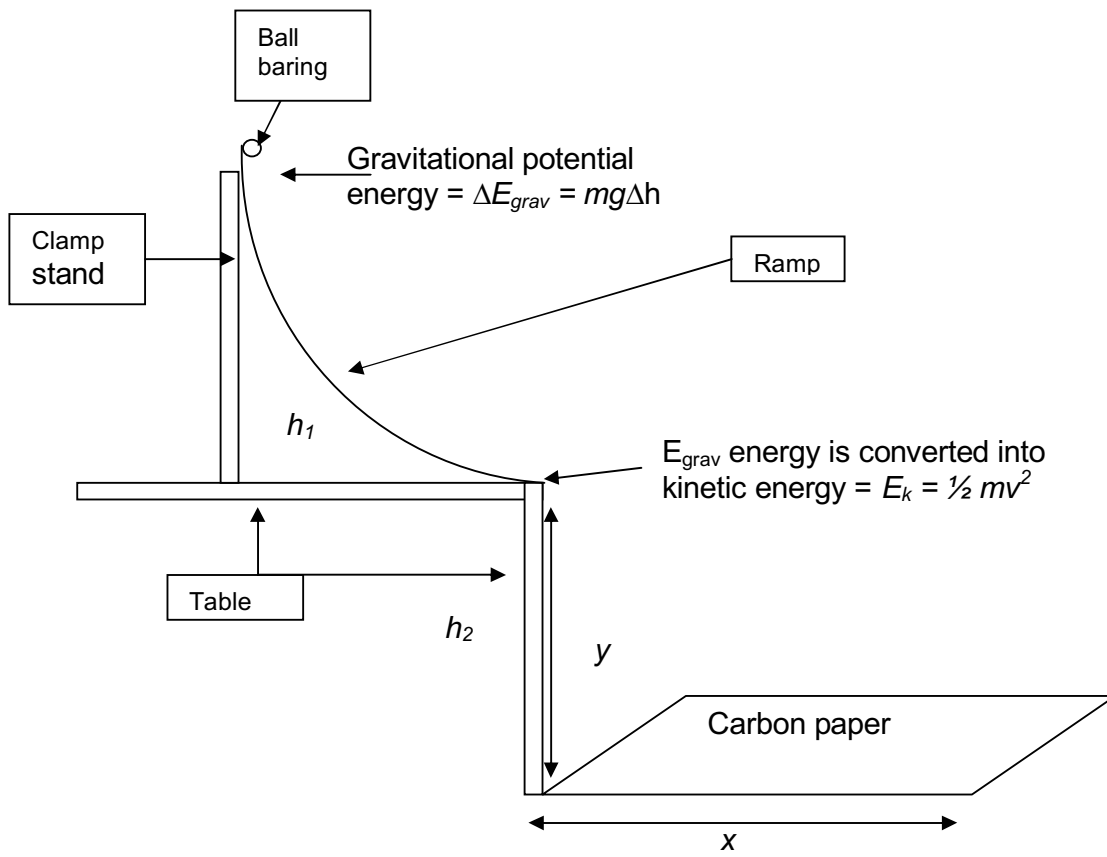
Equipment list:

- 1 Meter ruler stating centimetres and millimetres.
- Long length carbon paper
- 1 medium-sized ball bearing
- Plastic ramp/run
- 4 x A3 Plain Paper
- Pencil
- Weighing scales (in Newtons)
- Clamp stand, boss and clamp
- Spirit level
- Masking tape
- 4 Thick books adjust the height.
- Table drawn with the correct values to record the results

Procedure

1. Set up the apparatus shown in the diagram using the clamp stand boss and clamp, and the plastic ramp on top of a table.
2. Place the carbon paper on the floor, placing 4 x A3 sheets of plain paper underneath the carbon paper.
3. Make sure that the top of the ramp curves downwards towards the edge of the table, so the end of the ramp meets the edge of the table.
4. Weigh the ball bearing using the weighing scales (in Newtons) and note down the weight.
5. Make sure that the ramp is secure, use masking tape if needed and measured accurately using the meter ruler. The height (h^1) is 30 cm from the table to the point of drop – this will be the starting height.
6. Position the ball bearing on the ramp where the bottom of the ball bearing touches the 30 cm point. Steady the ball with your finger until release.
7. Once ready, release the ball bearing down the ramp, making sure that it hits the carbon paper. Repeat this 3 times. Measure the range from the furthest point of contact on the paper to the edge of the table.
8. Change the height of the ramp so the height (h^1) is 40cm from the table to the point of drop by adjusting the clamp and lift it to the 40 cm on the meter ruler. Place the ball at the point where it was last dropped. Repeat step 7.
9. Repeat step 8, positioning the height as 50cm, 60cm, 70cm and 80cm.

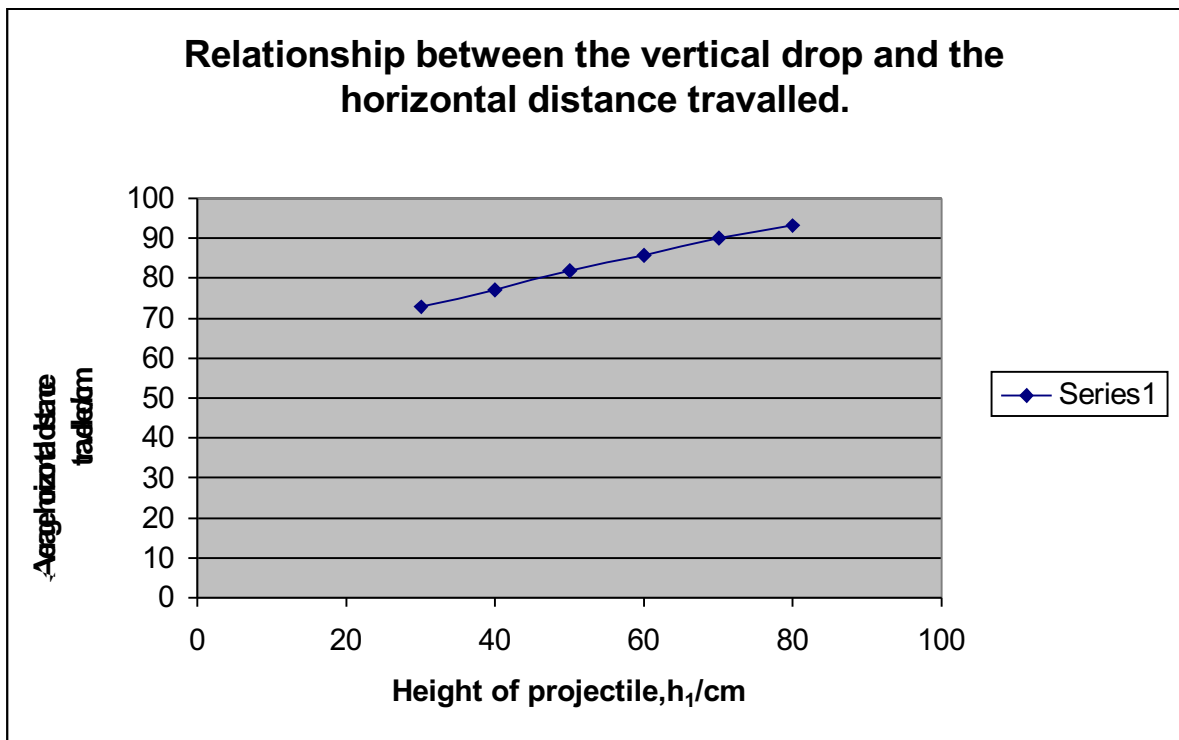
Diagram of apparatus



Results

Vertical distance from the horizontal launching level (h_1) $\pm 1.0\text{mm}$	Total horizontal distance travelled (range) $\pm 1.0\text{mm}$				Average/cm
	Trial 1	Trial 2	Trial 3	Trial 4	
30	72.50	71.30	73.40	74.00	72.80
40	76.60	78.50	76.70	77.30	77.20
50	82.20	81.60	82.00	81.40	81.80
60	85.70	85.40	85.80	85.70	85.65
70	89.90	90.50	90.40	89.70	90.13
80	93.50	92.80	92.90	93.30	93.12

Graph



Analysis

As you can see from the graph, as h_1 is increasing, the horizontal distance also increases. The graph is more or less a straight line because the horizontal distance travelled by the ball during in each interval should more or less be around the same. However it didn't show that the y -component is directly proportional to the x -component. This could be of the inaccuracy of the equipment, measurement and air resistance when in travelling in the air and friction on the ramp. The accuracy of the meter ruler is quite poor when it comes to measuring the point at which the ball has landed. Therefore the uncertainties would be measuring precisely the point at which the ball had landed. It will be more or less $\pm 1\text{mm}$.

The position for releasing the ball during is another issue. If the ball is being released from a higher or lower position compared to the previous test, it will have an affect on the initial velocity when leaving the ramp. Higher velocity will result in a bigger horizontal component and therefore uncertainties would be more or less $\pm 1\text{mm}$. The total uncertainty would be $\pm 2\text{mm}$.

The plastic ramp also creates a problem. The end of the ramp is difficult to maintain a precise horizontal position because of the bendiness of the plastic ramp. This is an important factor because it has an affect on the initial velocity and therefore will change the results. It will create a vertical acceleration if it bends resulting in an increase in the horizontal component.

In theory, we have assumed that the air through which the projectile moves has no effect on its motion, a reasonable assumption at low speeds. However, for a greater speed, the disagreement between calculations and the actual motion of the projectile can be large because the air opposes the motion of the projectile. So, a bigger h_1 means the projectile will be in the air for longer; therefore the air resistance will affect the projectile more and consequently will reduce horizontal distance travelled. I believe this is the reason that the line on the graph isn't directly proportional to the x and y components. Therefore in the absence of air resistance, I believe that the graph produced would be directly proportional to both x and y components.

In order to calculate the theoretical range, we have to find the horizontal and vertical component separately. I can use the following equations to find out both of the horizontal and vertical components.

Vertical Component:

GPE lost = KE gained

$$mgh = \frac{1}{2}mv^2$$

$$gh = \frac{1}{2}v^2$$

$$V = \sqrt{2gh}$$

v = velocity

m = mass (cancelled out)

g = gravity

h = height of the drop on slope

Horizontal Component

$$s = ut + \frac{1}{2}gt^2$$

$$s = \frac{1}{2}gt^2$$

$$t^2 = 2s / g$$

$$t = \sqrt{(2s / g)}$$

u = initial velocity, it is zero because the particle initially has no vertical velocity.

s = drop height (from top of the slope to sand tray)

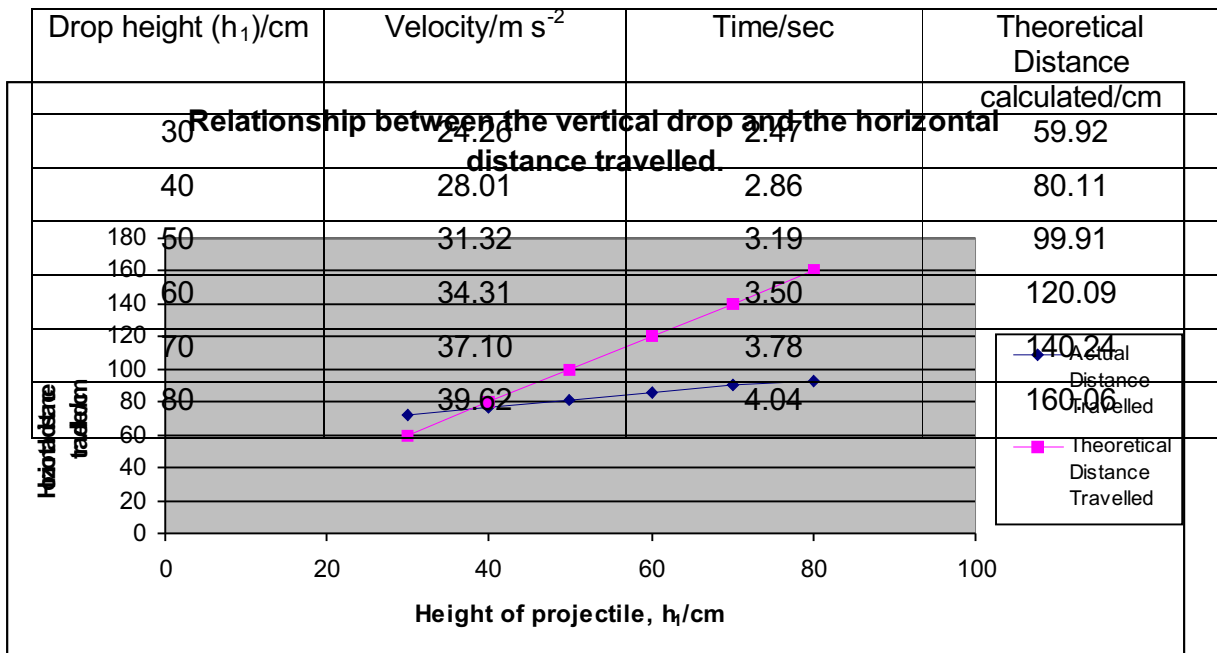
t = time between the moment it leaves the ramp and the impact on the sand.

g = acceleration due to gravity which is about 9.81 (assumed to be constant)

Now finally after we found the velocity and the time, it would be very easy to calculate the distance using the velocity equal distance divided by time equation, which can be re-arranged to equal: -

$$\text{Distance} = \text{Velocity} \times \text{Time}$$

I have used the equations above to produce the following table:



Below is a graph that shows the theoretical values and my actual values

Evaluation

There are different things that I could change to improve the accuracy of the experiment. I can put a gate as a releasing barrier at the point of dropping the ball bearing to ensure that it is being released at the same point every time. This can also help to keep the initial velocity leaving the ramp the same for

each trial. Strong tapes and sticky blue -tac can be used to keep the end of the ramp in position and to maintain at horizontal level. This helps to provide uniform acceleration for the vertical component.

In order to make the results as accurate as possible, we can try to eliminate all the uncertainties. However, some of the limitations cannot be eliminate. Air resistance is always present and is a force that always opposes the action of the ball baring. However, I can change the type of motion of the ball baring. I can use objects that slides rather than rolls down. However, it is hard to make the same object slide and not roll at the same time, whilst minimizing any friction that may occur. The ball baring is always going to roll down and produce a spinning effect. If a spinning effect occurs then energy is being released and wasted in the surroundings and perhaps not reaching the velocity that it could. If I use another object to slide down, the air resistance would be different and therefore it will only result in increasing the uncertainties due to an increase of friction on the ramp. But I can still carry further investigations in these criteria. Another interesting point would be changing the vertical acceleration of the ball baring. However, this is impossible for me since we will have to change the gravity of the surrou ndings for the ball baring to accelerate faster then 9.81ms^{-2} .

In this investigation, I have taken account of h_1 as my variable and all other factors are being fixed and so remain constant. I can also investigate h_2 if I will do this experiment again. It is different from h_1 because the ramp will always be in a fixed position and so the angle of the ramp will not change. So, the initial velocity will always be the same.

Conclusion

The conclusion to this experiment is that as the vertical projecting height increases, the horizontal distance travelled also increases. However, it is the air resistance and friction of the ramp that limits the projectile motion. From my results, it shows that air resistance has little effect on the ball baring if the time spent in the air is less, when h_1 is at a low height. However, if the ball baring travels in the air for longer, when h_1 is at a higher level, the affect of air resistance is applied for longer when the projectile leaves the ramp and affects the horizontal distance travelled. From my results, I can conclude that my investigation supports my hypothesis and disproves my null hypothesis. However, this also proves that air resistance and friction is a big limiting factor.