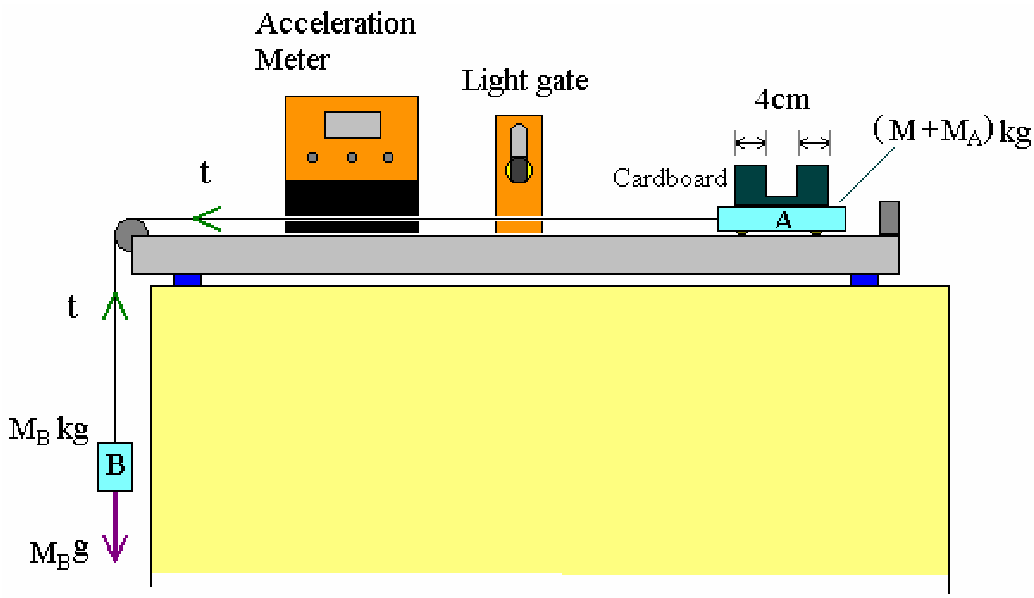
Investigating Force, Mass and Acceleration using a Trolley

Aim of experiment:

Using simple trolley experiments, take suitable measurements from them to find the Mass of the trolley, the fraction force acting in the system and to investigate any other effects found when using apparatus.



Let M = the mass of the trolley (A), M_A = Mass added onto the trolley

M_B = the mass of the hanging mass (B)

t = Tension in the string

g = Gravitational force = 9.8 N

R = the resistive force acting in the system.

i.e. Fraction between the track and the trolley, fraction force in the pulley and air resistance.

The Diagram above shows the setup of apparatus for the particular experiments. The acceleration meter measures the acceleration of the system by measuring the time taken through each of the 4cm cardboard and the time interval (t) between the two cardboards. The speed (u, v) through each of them is calculated and the acceleration is calculated using the formulae.

$a = (v - u)/ t$

Two sets of experiments are carried out:

First experiment:

0.09 kg of mass (M_A) is placed on the trolley and the hanging mass (M_B) is 0.01 kg at the start. The acceleration of the system is measured. Then 0.01 kg of mass is taken from the trolley each time and added to the hanging mass and acceleration of the system is measured. The experiment is carried on as the same procedure until $M_A = 0$ kg and $M_B = 0.1$ kg

In this case $M_A + M_B$ is always equal to 0.1 kg and the mass of the system is always equals to $M_A + M_B + M = (M + 0.1)$ kg

Using $F - R = ma$ where a = acceleration, m = mass

Acceleration of A: $t - R = (M + M_A)a$

B: $M_B g - t = M_B a$ $M_B g - M_B a = t$

$\therefore$ $M_B g - M_B a - R = (M + M_A)a$

$M_B g - R = a (M + M_A + M_B)$

$a = (M_B g - R) / (M + M_A + M_B)$

$a = (M_B g - R) / (M + 0.1)$

The following table represents the results from the above experiment.

Hanging Mass (M_B)/kg	Force / ($M_B g$)N	Acceleration$_1$ / ms^{-2}	Acceleration$_2$ / ms^{-2}	Acceleration$_3$ / ms^{-2}	Average Accel. / ms^{-2}
0.01	0.10	0.13	0.13	0.13	0.13
0.02	0.20	0.32	0.31	0.32	0.32
0.03	0.29	0.51	0.50	0.51	0.51
0.04	0.39	0.72	0.73	0.71	0.72
0.05	0.49	0.90	0.92	0.90	0.91
0.06	0.59	1.14	1.11	1.11	1.12
0.07	0.69	1.35	1.37	1.35	1.36
0.08	0.78	1.52	1.55	1.53	1.53
0.09	0.88	1.74	1.70	1.72	1.72
0.10	0.98	1.94	1.93	1.97	1.95

The straight-line graph on the previous page shows that the Force acts on the trolley is proportional to the acceleration of the trolley.

Let **F** = ($M_B g$) the force acting on the trolley
A = acceleration
$\Sigma A = 10.26$ $\Sigma \mathbf{F} = 5.390$ $\Sigma AF = 7.167$ $\Sigma \mathbf{A}^2 = 13.91$
n = 10

The equation of the linear regression of F on A is:
F = a + bA
Where $b = \mathbf{S}_{AF} / \mathbf{S}_{AA}$

$$a = \bar{\mathbf{F}} - \mathbf{b}\,\bar{\mathbf{A}}$$

$\mathbf{S}_{AF} = \Sigma \mathbf{AF} - (\Sigma F \Sigma \mathbf{A}) / \mathbf{n}$
$= 7.167 - (5.390 \times 10.26) / 10$
$= 1.638$
$\mathbf{S}_{AA} = \Sigma \mathbf{A}^2 - (\Sigma A)^2 / \mathbf{n}$
$= 13.91 - (10.26)^2 / 10$
$= 3.388$

$b = 1.638 / 3.388 = 0.483$ $a = 5.390 / 10 - 0.483(10.26 / 10) = 0.0434$
Regression: $F = 0.483A + 0.0434$

If the force $M_B g$ is made into the subject on the equation obtained from the model.
$a = (M_B g - R) / (M + 0.1)$
$M_B g - R = (M + 0.1)\,a$
$M_B g = (M + 0.1)\,a + R$
$F = M_B g$ $\therefore$ $F = (M + 0.1)\,A + R$

According to the regression equation.
M + 0.1 = 0.484
M = 0.383
∴ The mass of trolley is approximately 0.38 kg (2 S. F.)

R = 0.0420
∴ The resistive force acting on the system is approximately 0.043 N (2 S. F.)

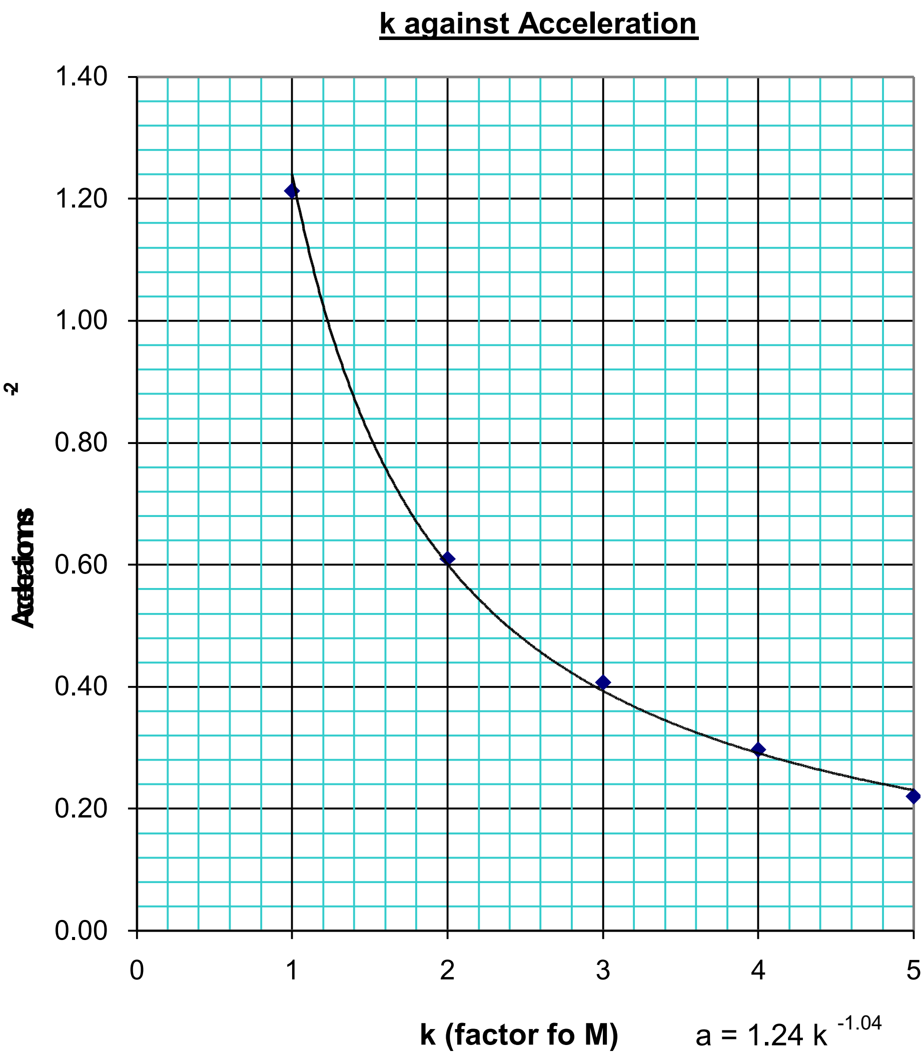
Second Experiment:

On this experiment, the Mass of the hanging mass (M_B) is kept constant. And the Mass of the trolley is varied from 1M kg to 5M kg and the acceleration is measured on each mass. M = mass of trolley
The hanging mass used is 0.06 kg which acts a force of (0.06g) N on the system.

Using the equation obtained before: $a = (M_B g - R) / (M + M_A + M_B)$
Let k = factor of M $1 \leqq k \leqq 5$
$a = (M_B g - R) / (kM + M_B)$
$a = (0.06g - R) / (kM + 0.06)$

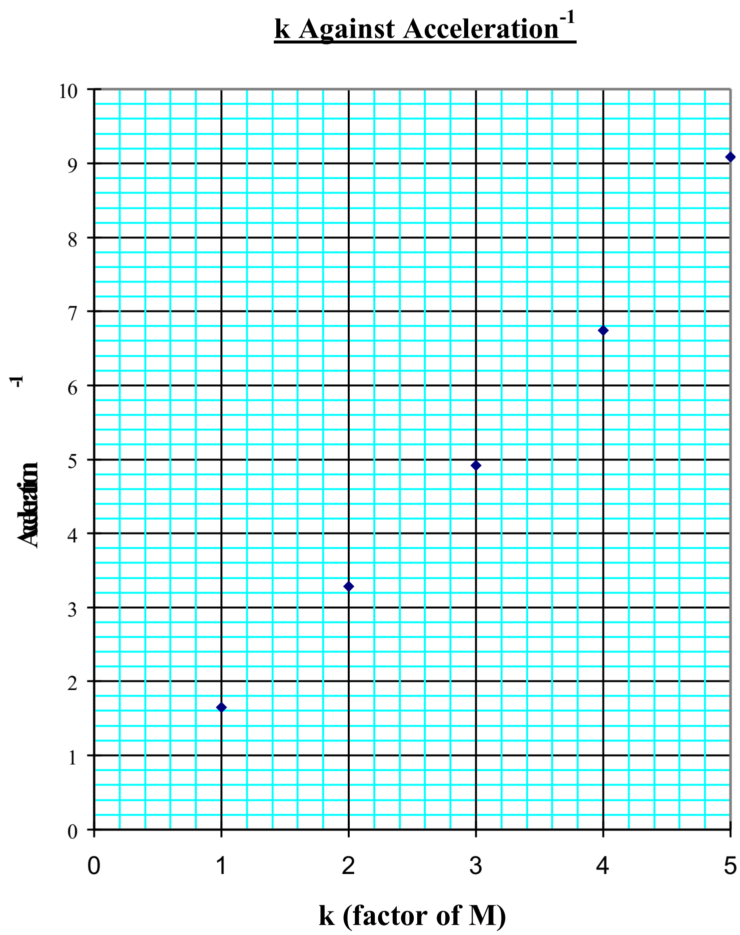
The Table below shows the results obtained form this experiment.

k	Mass/(kM)kg	$Acceleration_1$ / ms^{-2}	$Acceleration_2$ / ms^{-2}	$Acceleration_3$ / ms^{-2}	Average Accel / ms^{-2}
1	1M	1.22	1.22	1.20	1.21
2	2M	0.61	0.61	0.61	0.61
3	3M	0.41	0.40	0.41	0.41
4	4M	0.30	0.29	0.30	0.30
5	5M	0.22	0.23	0.21	0.22



The Graph show that **a** is approximately equals to 1.24 k^{-1}. The Acceleration is inversely proportional to k and as k doubles, the acceleration halves.

In order to prove that $a = (0.06g - R) / (kM + 0.06)$ supports the graph and the data from the second experiment. A graph with inverse acceleration (1/**a**) against k is plotted.



Graph showing 1/a against k

By making a into 1/a in the equation.
The equation of the graph could be formed.

$$a = (0.06g - R) / (kM + 0.06)$$
$$1/a = (kM + 0.06) / (0.06g - R)$$
$$1/a = kM/ (0.06g - R) + 0.06 / (0.06g - R)$$
$$1/a = k (M/ (0.06g - R) + 0.06 / (0.06g - R)$$

A linear equation is formed and would give a straight line if g, M and R are constant. However, the graph does not fit perfectly to the equation due to the slightly curve of the line. The gradient of the line is increasing slightly as k increase. This could be explained if one of the factor g, M or R depends on the value of k. Assumed the variation of g is so small that can be neglected and M is constant and do not affect by k, then R is the factor that causes the increase of gradient and it increases as the value of k increases.

Evaluation

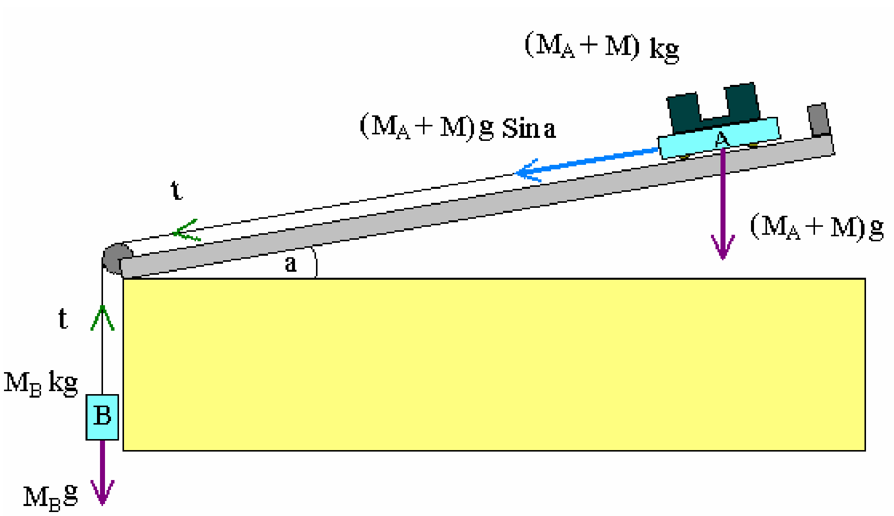
From the first experiment, the mass of the trolley and the resistive force in the system is obtained but there are a few numbers of systematic errors caused by the apparatus which can make the result inaccurate.
The second experiment show that the resistance does not stay constant as the mass of the trolley change, this can cause slight error in the first experiment as well.

Errors:

I) The string used in the experiment must not be 100% inextensible therefore a small fraction of the force will be wasted as the result of extending the string and According to Hooke's law force is directly proportional to the extension of the string which means as the tension on the string doubles, the extension will also doubles, therefore the wasted force will doubles.

II) The track might be slightly incline due to the table or the floor which the track is not perpendicular to the gravitational force of earth and create an unwanted force in the system.

For example



The unwanted force ($(M_A + M)g \sin a$) will increase the acceleration of the trolley.

III) The light gate used in the experiment might carry out small errors. The sensitivity of the light sensor and the width of the hole for light to enter the light gate might vary the length of cardboard that it is measuring from + or – the radius of the hole.

IV) The Acceleration meter could also carry out errors during its calculation of acceleration. As the acceleration increases the time interval it measures decreases and the percentage error increases. E.g. If the highest accuracy that it can measure is 0.05 s. A time interval of 1 s will have ±5 % error but if the time interval decreases to 0.1 s the maximum error will increase to ±50 %.

V) C The mass used in the experiment can also cause small errors. I.e. 10gram mass might have an error of +/– 0.1 gram. Giving a percentage error of 1%.

Errors calculations

The table below shows that as the acceleration increase, the error in acceleration increase

Acceleration +/– = **$\text{Acceleration}_{max}$ – $\text{Acceleration}_{min}$**

$\text{Acceleration}_{max}$ / ms^{-2}	Acceleration_2 / ms^{-2}	$\text{Acceleration}_{min}$ / ms^{-2}	Average Accel / ms^{-2}	Acceleration +/- ms^{-2}
0.13	0.13	0.13	0.13	0.00
0.32	0.32	0.31	0.32	0.01
0.51	0.51	0.5	0.51	0.01
0.73	0.72	0.71	0.72	0.02
0.92	0.9	0.9	0.91	0.02
1.14	1.11	1.11	1.12	0.03
1.37	1.35	1.35	1.36	0.02
1.55	1.53	1.52	1.53	0.03
1.74	1.72	1.7	1.72	0.04
1.97	1.94	1.93	1.95	0.04

A same graph for the first experiment is plotted but with error bars.

The maximum and minimum of the mass of trolley and the resistive force is calculated.
The _______ line represent the maximum regression F = 0.4933A+ 0.0439
M + 0.1 = 0.493 R = 0.0439 N
M = 0.393 kg

The _______ line represent the minimum regression F = 0.4739A + 0.0425
M + 0.1 = 0.474 R = 0.0425 N
M = 0.374 kg

0.393 – 0.383 = 0.01 0.0439 – 0.0434 = 0.001
0.374 – 0.383 = -0.01 (2d.p.) 0.0425 – 0.0434 = -0.001 (3d.p.)

Therefore the mass of the trolley is approximately 0.38 +/– 0.01

Resistive force is approximately 0.043 N +/– 0.001

The final result I obtain is only an approximate value due to the fact that there is uncertain errors caused by the light gate, the inclination of the track and the variation of the mass used. In the second experiment the resistive force of the system is proved to be inconstant as the mass of the trolley increase and this could cause errors on the results I obtained from the first experiment because the mass of the trolley is not constant and I assumed the resistive force to be constant in the calculation.