Insolation refers to the deposition of radiant energy as heat into an absorbing body.

If the body is a planet, first obtain the declination of the Sun as seen from a pole of a planet, use equatorial coordinates. Let a planet have obliquity β and the Sun have longitude λ and right ascension α as viewed from the planet. From celestial to equatorial coordinate transformation,

$$\sin\delta = \sin\beta\sin L, \tag{1}$$

where δ is the Sun's declination. In general, the longitude of the sun (L) will not be zero at the vernal equinox $(\theta = 0)$, but will be offset by an angle ϕ. perihelion for the earth, for example, occurs in January, while the vernal equinox is not until March. So it must be remembered that

$$\theta \equiv \phi + L. \tag{2}$$

We need to find the normal component of radiation at the north pole. But this flux will be simply

$$\Phi = \Phi_0 \sin\delta, \tag{3}$$

since the angular altitude of the Sun from the horizon is given by δ. Using

$$\Phi_0 = \frac{P}{4\pi r^2} \tag{4}$$

and plugging (4) into (1) and (4) gives

$$\Phi = \frac{P}{4\pi r^2}\sin\beta\sin L. \tag{5}$$

Now, we are interested in finding the average of this flux over a full orbit. If the orbit is eccentric, the time required to travel an orbital distance $d\theta$ is not constant, but related to *dt* according to

$$\frac{d\theta}{dt} = \frac{h}{r^2} = \frac{na^2\sqrt{1-e^2}}{r^2} \tag{6}$$

$$dt = \frac{r^2}{na^2\sqrt{1-e^2}}\, d\theta = \frac{Tr^2}{2\pi a^2\sqrt{1-e^2}}\, d\theta. \tag{7}$$

From (2),

$$dL = d\theta, \tag{8}$$

which makes physical sense, since *dt* depends not on how the zero point for angular position is chosen, but on what the orbital distance is at the relevant position (the r^2 dependence). Equation (7) therefore becomes

$$dt = \frac{Tr^2}{2\pi a^2\sqrt{1-e^2}}\, dL. \tag{9}$$

To find the time-average flux, simply take

$$\langle \Phi \rangle = \frac{1}{T}\int \Phi\, dt \tag{10}$$

over the orbit. Note, however, that when $L < 0$ or $L > \pi$ (between autumnal equinox and vernal equinox), the north pole will be facing away from the sun and will therefore receive zero flux. The only nonzero contribution to the flux in (?) therefore occurs for $0 < L < \pi$.

$$\begin{aligned} \langle \Phi \rangle &= \frac{1}{T}\int \Phi\, dt \\ &= \frac{1}{T}\frac{P}{4\pi}\frac{T}{2\pi a^2\sqrt{1-e^2}}\int_0^\pi \frac{r^2}{r^2}\sin\beta\sin L\, dL \\ &= \frac{P}{4\pi a^2}\frac{\sin\beta}{2\pi\sqrt{1-e^2}}\int_0^\pi \sin L\, dL \end{aligned}$$

$$
\begin{aligned}
&= \frac{S\sin\beta}{2\pi\sqrt{1-e^2}}[-\cos L]_0^{\pi} \\
&= -\frac{S\sin\beta}{2\pi\sqrt{1-e^2}}[(-1)-(1)] = \frac{S\sin\beta}{\pi\sqrt{1-e^2}}.
\end{aligned}
$$