The Trifilar Suspension

Tutor : Dr. Yan
Name : Manilka Abeysuriya
Course : Aeronautical Engineering
Group : A – 1
Date : 27 / 01 / 2003

Title :

Determination of moment of inertia of a uniform rectangular bar and a connecting rod using the trifilar suspension, and by swinging the connecting rod as a compound pendulum.

Introduction :

Moment of Inertia can be described as a measure of "unwillingness to change the current motion" of a certain body of mass. In the experiment, the main objective is to find the moment of inertia of a uniform body and an irregular shaped body.
Toward achieving this
First the center of mass of the bar and the connecting rod was found by balancing them on a knife-edge. Then the bar was placed on the trifilar suspension which is a circular platform suspended by three equally spaced wires of equal length, such that the center of mass of the rod is over the center of the circular platform. Then the whole system is given a small angular displacement, and the periodic time for the oscillations is determined by measuring the time taken for 20 oscillations. By using the equation 1 the moment of inertia of the bar about the axis through its center of mass can be calculated. The same procedure is followed for the connecting rod and its moment of inertia about the axis passing through its center of mass was found.
After that the connecting rod was suspended by a one end from a knife-edge, and given a small angular displacement. The time taken for 20 oscillations was measured and the periodic time was found. By using equation 2 the moment of inertia of the rod about the axis through the point of suspension can be found. The same procedure was followed, by suspending the rod by the other end. Using equation 2, moment of inertia of the rod about the axis through the point of suspension can be found. Finally by using the parallel axis theorem the moment of inertia of the rod about the axis through the center of mass can be found.

Results :

For the uniform rod (using trifilar suspension)	$I = (77.4 \pm 2.8)10^{-5}\,kgm^2$
For the uniform rod (theoretically)	$I = (108.0 \pm 0.0553)10^{-5}\,kgm^2$
For the connecting rod (using trifilar suspension)	$I = (185.0 \pm 1.2)10^{-5}\,kgm^2$
For the connecting rod (1st suspension)	$I = (17.0 \pm 1.7)10^{-3}\,kgm^2$
For the connecting rod (2nd suspension)	$I = (295.0 \pm 4.0)10^{-5}\,kgm^2$

Discussion :

The equation 1 is accurate only for oscillations with small angular displacements. If this displacement is considerably large an error could occur in the final results due to this reason. And also when using the trifilar suspension the center of mass of the object under consideration should be right over the platform. Other wise a additional moment of inertia of the system will cause a increase of the result, actually expected. When measuring the time an error is always possible in the measurement due to differences in human reaction time.

Analysis of Error,

When I = 0 and M = 0,

From equation 1, $I_0 = \left(\frac{T}{2\pi}\right)^2 \frac{r^2 g M_0}{L}$

Error in I_0, $\frac{\delta I_0}{I_0} = \sqrt{2\left(\frac{\delta T}{T}\right)^2 + 2\left(\frac{\delta r}{r}\right)^2 + \left(\frac{\delta M_0}{M_0}\right)^2 + \left(\frac{\delta L}{L}\right)^2}$

$$= \sqrt{2\left(\frac{0.01}{1.980}\right)^2 + 2\left(\frac{1\times 10^{-4}}{7.75\times 10^{-2}}\right)^2 + \left(\frac{0.001}{1.125}\right)^2 + \left(\frac{0.001}{1.705}\right)^2}$$

$= 7.62 \text{ x}10^{-3}$

$\delta I_0 = 7.62 \text{ x}10^{-3} \text{ x } 3.67 \times 10^{-3} \, kgm^2$

$\delta I_0 = 2.80 \times 10^{-5} \, kgm^2$

For the uniform rod,

Let m = (M + M_o)

Then, m = (1125 + 1954.8) x 10^{-3} kg

= 3.080 kg

Error in m is, $(\delta m)^2 = (\delta M)^2 + (\delta M_0)^2$

$$(\delta m)^2 = \{(0.0001)^2 + (0.001)^2\} kg$$

= 1.01 x 10^{-6} kg

From equation 1,

$$I = \left(\frac{T}{2\pi}\right)^2 \frac{r^2 g \left(M + M_0\right)}{L} - I_0$$

Let $I_1 = \left(\frac{T}{2\pi}\right)^2 \frac{r^2 g \left(M + M_0\right)}{L}$

Error in I_1, $\frac{\delta I_1}{I_1} = \sqrt{2\left(\frac{\delta T}{T}\right)^2 + 2\left(\frac{\delta r}{r}\right)^2 + \left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta L}{L}\right)^2}$

$$= \sqrt{2\left(\frac{0.01}{2.057}\right)^2 + 2\left(\frac{1\times 10^{-4}}{7.75\times 10^{-2}}\right)^2 + \frac{1.01\times 10^{-6}}{(3.080)^2} + \left(\frac{1\times 10^{-3}}{1.705}\right)^2}$$

$= 5.1\times 10^{-5}$ kgm^2

$$\delta I_1 = 5.1 \text{ x}10^{-5} \text{ x } 11.41 \times 10^{-3} kgm^2$$
$$= 6.0 \times 10^{-7} kgm^2$$

Error in I ,

$$\delta I = \sqrt{(\delta I_1)^2 + (\delta I_0)^2}$$
$$\delta I = \sqrt{(6.0 \times 10^{-7})^2 + (2.80 \times 10^{-5})^2} \text{ kgm}^2$$
$$= \underline{2.8 \times 10^{-5} kgm^2}$$

For the theoretical calculation of I for the uniform rod,

From equation 2,

$$I = \frac{M}{12}\left(l^2 + w^2\right)$$

Let $p = l^2 + w^2 = (0.2545^2 + .0385^2)\ m^2$

Then error in p, $\delta p = \sqrt{4(\delta l)^4 + 4(\delta w)^4}$

$$\delta p = \sqrt{4(1 \times 10^{-4})^4 + 4(1 \times 10^{-4})^4}\ m^2$$
$$\delta p = 3 \times 10^{-8}\ m^2$$

Now error in I, $\left(\frac{\delta I}{I}\right)^2 = \left(\frac{\delta M}{M}\right)^2 + \left(\frac{\delta p}{p}\right)^2$

$$\left(\frac{\delta I}{I}\right) = \sqrt{\left(\frac{1 \times 10^{-4}}{1.948}\right)^2 + \left(\frac{3 \times 10^{-8}}{0.062}\right)^2}$$
$$\delta I = 5.12 \times 10^{-5} \times 10.8 \times 10^{-3} \text{ kgm}^2$$
$$= \underline{5.53 \times 10^{-7} kgm^2}$$

For the connecting rod,
When using the trifilar suspension,

Let $m = (M + M_0)$

Then, $m = (1848.52 + 1954.8) \text{ x } 10^{-3}$ kg

$= 3.8033$ kg

Error in m is, $(\delta m)^2 = (\delta M)^2 + (\delta M_0)^2$

$$(\delta m)^2 = \{(0.0001)^2 + (0.001)^2\} kg$$
$$= 1.0001 \text{ x } 10^{-6} \text{ kg}$$

From equation 1,

$$I = \left(\frac{T}{2\pi}\right)^2 \frac{r^2 g (M + M_0)}{L} - I_0$$

Let $$I_1 = \left(\frac{T}{2\pi}\right)^2 \frac{r^2 g\,(M + M_0)}{L}$$

Error in I_1,
$$\frac{\delta I_1}{I_1} = \sqrt{2\left(\frac{\delta T}{T}\right)^2 + 2\left(\frac{\delta r}{r}\right)^2 + \left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta L}{L}\right)^2}$$

$$= \sqrt{2\left(\frac{0.01}{2.921}\right)^2 + 2\left(\frac{1\times 10^{-4}}{7.75\times 10^{-2}}\right)^2 + \frac{1.0001\times 10^{-6}}{(3.8033)^2} + \left(\frac{1\times 10^{-3}}{1.705}\right)^2}$$

$$= 5.2\times 10^{-3}\ \text{kgm}^2$$
$$\delta I_1 = 5.2\times 10^{-3}\times 22.17\times 10^{-3}\ kgm^2$$
$$= 1.2\times 10^{-4}\ kgm^2$$

Error in I,
$$\delta I = \sqrt{(\delta I_1)^2 + (\delta I_0)^2}$$
$$\delta I = \sqrt{(1.2\times 10^{-4})^2 + (2.80\times 10^{-5})^2}\ \text{kgm}^2$$
$$= \underline{1.2\times 10^{-4}\ kgm^2}$$

Compound pendulum method for con-rod.

When $l = 236$ mm,
Error in I,
$$\frac{\delta I}{I} = \sqrt{2\left(\frac{\delta T}{T}\right)^2 + \left(\frac{\delta M}{M}\right)^2 + \left(\frac{\delta l}{l}\right)^2}$$
$$= \sqrt{2\left(\frac{0.01}{1.051}\right)^2 + \left(\frac{1\times 10^{-5}}{1.8482}\right)^2 + \left(\frac{1\times 10^{-3}}{0.236}\right)^2}$$
$$\delta I = 1.4\times 10^{-2}\times 120\times 10^{-3}\ kgm^2$$
$$= 1.7\times 10^{-3}\ kgm^2$$

Let $Y = Md^2$, $$\left(\frac{\delta Y}{Y}\right) = \sqrt{\left(\frac{\delta M}{M}\right)^2 + 2\left(\frac{\delta d}{d}\right)^2}$$

$$= \sqrt{\left(\frac{1\times 10^{-5}}{1.8482}\right)^2 + 2\left(\frac{1\times 10^{-3}}{0.236}\right)^2}$$
$$\delta Y = 3.59\times 10^{-5}\times 0.103\ kgm^2$$
$$= 3.70\times 10^{-5}\ kgm^2$$

Error in I_{yy}, $$\delta I_{yy} = \sqrt{(\delta I)^2 + (\delta Y)^2}$$

$$\delta I_{yy} = \sqrt{(1.7\times 10^{-3})^2 + (3.70\times 10^{-5})^2}$$

$$\delta I_{yy} = 1.7\times 10^{-3}\, kgm^2$$

When l = 100 mm,
Error in I,

$$\frac{\delta I}{I} = \sqrt{2\left(\frac{\delta T}{T}\right)^2 + \left(\frac{\delta M}{M}\right)^2 + \left(\frac{\delta l}{l}\right)^2}$$

$$= \sqrt{2\left(\frac{0.01}{1.02}\right)^2 + \left(\frac{1\times 10^{-5}}{1.8482}\right)^2 + \left(\frac{1\times 10^{-3}}{0.100}\right)^2}$$

$$\delta I = 2.9\times 10^{-4}\times 48.0\times 10^{-3}\, kgm^2$$

$$= 1.4\times 10^{-5}\, kgm^2$$

Let Y=Md^2, $\left(\frac{\delta Y}{Y}\right) = \sqrt{\left(\frac{\delta M}{M}\right)^2 + 2\left(\frac{\delta d}{d}\right)^2}$

$$= \sqrt{\left(\frac{1\times 10^{-5}}{1.8482}\right)^2 + 2\left(\frac{1\times 10^{-3}}{0.100}\right)^2}$$

$$\delta Y = 3.59\times 10^{-5}\times 0.103\, kgm^2$$

$$= 3.70\times 10^{-5}\, kgm^2$$

Error in I_{yy},

$$\delta I_{yy} = \sqrt{(\delta I)^2 + (\delta Y)^2}$$

$$\delta I_{yy} = \sqrt{(1.4\times 10^{-5})^2 + (3.70\times 10^{-5})^2}\; kgm^2$$

$$\delta I_{yy} = 4.0\times 10^{-5}\, kgm^2$$

Appendix :

For the trifilar suspension the periodic time is given by,

$$T = 2\pi\sqrt{\frac{L}{r^2 g}\left(\frac{I_0 + I}{M_0 + M}\right)} \qquad \text{Equation 1}$$

where,

L	-	length of the suspension wires,
r	-	radius from the center of the platform to the attachment points of the suspension wires,
I_o	-	moment of inertia of platform,
M_o	-	mass of platform,

I	-	moment of inertia of body,
M	-	mass of body,
G	-	acceleration due to gravity.

Raw data,

L	1.705 m
r	0.0775 m
M_o	1.125 kg

For the oscillations of the trifilar suspension,

T_1	1.935 s
T_2	1.920 s
T_3	1.934 s
Average - T	1.930 s

From Equation 1,

$$1.930\ (s) = 2\pi\sqrt{\frac{1.705\ (m)}{0.0775^2 (m^2) \times 9.81\ (ms^{-2})}\left(\frac{I_0 + 0}{1.125\ (kg) + 0}\right)}$$

$$I_0 = \left(\frac{1.930}{2\pi}\right)^2 \frac{0.0775^2 \times 9.81 \times 1.125}{1.705} kgm^2$$

$$\underline{I_0 = 3.67 \times 10^{-3} (kgm^2)}$$

Dimensions for uniform rod,

Length	254.5mm
Width	38.5mm
Mass	1954.8g

For the oscillations of the system rod and the trifilar suspension,

T_1	2.056 s
T_2	2.050 s
T_3	2.066 s
Average - T	2.057 s

From Equation 1,

$$2.057\ (s) = 2\pi\sqrt{\frac{1.705\ (m)}{0.0775^{2}(m^{2})\times 9.81\ (ms^{-2})}\left(\frac{3.67\times 10^{-3}(kgm^{2}) + I}{1.125\ (kg) + 1954.8\times 10^{-3}(kg)}\right)}$$

$$I = \left\{\left(\frac{2.057}{2\pi}\right)^{2}\frac{0.0775^{2}\times 9.81\times 3080}{1.705} - 3.67\times 10^{-3}\right\}kgm^{2}$$

$$\underline{I = 7.74\times 10^{-3}\,kgm^{2}}$$

Theoretically,

$$I = \frac{M}{12}\left(l^{2} + w^{2}\right) \qquad \text{Equation 2}$$

$$I = \frac{1.9548\ (kg)}{12}\left(0.2545^{2} + 0.0385^{2}\right)kgm^{2}$$

$$I = 10.792\times 10^{-3}\,kgm^{2}$$

$$\underline{I = 10.8\times 10^{-3}\,kgm^{2}}$$

For the connecting rod,

Mass	1848.52 g
T_1	2.918 s
T_2	2.923 s
T_3	2.922 s
Average - T	2.921 s

Using Equation 1 for the system of trifilar suspension and the connecting rod,

$$2.921\ (s) = 2\pi\sqrt{\frac{1.705\ (m)}{0.0775^{2}(m^{2})\times 9.81\ (ms^{-2})}\left(\frac{3.67\times 10^{-3}(kgm^{2}) + I}{1.125\ (kg) + 1848.52\times 10^{-3}(kg)}\right)}$$

$$I = \left\{\left(\frac{2.921}{2\pi}\right)^{2}\frac{0.0775^{2}\times 9.81\times 2974}{1.705} - 3.67\times 10^{-3}\right\}kgm^{2}$$

$$\underline{I = 18.5\times 10^{-3}\,kgm^{2}}$$

When the connecting rod was suspended by a end let to swing the moment of inertia of the rod about the axis through the point of suspension is given by,

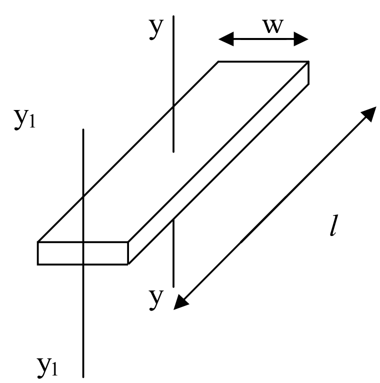
$$I = \frac{T^2}{4\pi^2}.M.g.l \text{ .... Equation 3}$$

Where,

T	-	periodic time
M	-	mass
G	-	acceleration due to gravity
l	-	distance from the point of suspension.

Using the parallel axes theorem,

$$I_{y1y1} = I_{yy} + Md^2$$
$$I_{yy} = I_{y1y1} - Md^2 \text{............Equation 4}$$



When $l = 236$ mm,

T_1	0.9980 s
T_2	1.084 s
T_3	1.072 s
Average - T	1.051 s

From equation 3,

$$I = \frac{1.051^2 (s^2)}{4\pi^2}.1.8452 \ (kg) \times 9.81 \ (ms^{-2}) \times 0.236 \ (m)$$

$$I = 119.7 \times 10^{-3}\ kgm^2$$

$$I = 120 \times 10^{-3}\ kgm^2$$

From Equation 4,

$$I_{yy} = \left(120 \times 10^{-3} - 1.8452 \times 0.236^2\right) kgm^2$$

$$\underline{I_{yy} = 17.0 \times 10^{-3}\ kgm^2}$$

When l = 100 mm,

T_1	0.9950 s
T_2	1.030 s
T_3	1.040 s
Average - T	1.022 s

From equation 3,

$$I = \frac{1.022^2 (s^2)}{4\pi^2} \times 1.8452\ (kg) \times 9.81\ (ms^{-2}) \times 0.100\ (m)$$

$$I = 47.97 \times 10^{-3}\ kgm^2$$

$$I = 48.0 \times 10^{-3}\ kgm^2$$

From Equation 4,

$$I_{yy} = \left(48.0 \times 10^{-3} - 1.8452 \times 0.100^2\right) kgm^2$$

$$\underline{I_{yy} = 29.5 \times 10^{-3}\ kgm^2}$$