## ***Title:***

Measuring Young's modulus of copper

## ***Aim:***

To study the stress/strain behavior of copper wire and estimate the Young's modulus of copper

## ***Apparatus:***

| | |
|---|---|
| Copper wire s.w.g.32 | about 4 m |
| G-clamp | ×1 |
| Wooden block | ×2 |
| Metre rule | ×4 |
| Pulley on clamp | ×1 |
| Micrometer screw gauge | ×1 |
| Hanger (0.01 kg) | ×1 |
| Slotted mass (0.05 kg) | ×8 |
| Slotted mass (0.1 kg) | ×6 |
| Slotted mass (0.2 kg) | ×4 |
| Slotted mass (0.5 kg) | ×1 |
| White label sticker | ×1 |
| Safety goggles | ×1 |
| Rubber tile | ×1 |

## ***Theory:***

When a force $F$ is applied to the end of a wire with cross-sectional area $A$ along its length, the tensile stress = $\frac{F}{A}$

If the extension of the wire is $\Delta l$, and its original length is $l_o$, the tensile strain = $\frac{\Delta l}{l_o}$

Under elastic conditions, a modulus of elasticity of a wire, called the Young modulus $E$, is defined as the ratio of the tensile stress applied to a body to the tensile strain produced. $E = \frac{stress}{strain} = \frac{F/A}{\Delta l / l_o}$ where $E$ is expressed in N $m^{-2}$ or Pascal (Pa).

$E$ is a constant when $\Delta l$ is small according to the Hooke's Law which stated that the stress applied to any solid is proportional to the strain it produces for small strain.

Therefore, when a material has a larger the value of $E$, it resists to the elastic deformation strongly and a large stress is required to produce a small strain. $E$ is thus a measure of the elastic stiffness of a material.

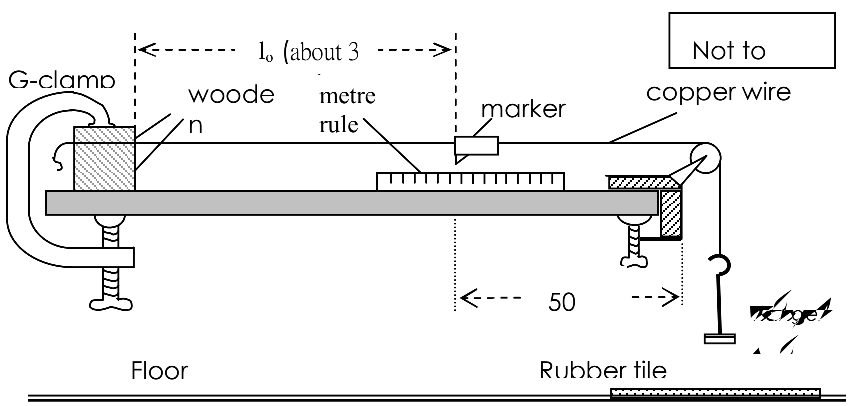
However, when the extension (deformation) of the wire is too large, beyond proportional limit, solid will no longer obey Hooke's law i.e. $E$ is no longer a constant.

As the stress further increases, beyond the elastic limit, the wire has a permanent extension that the wire is no longer elastic and it undergoes plastic deformation. The extension increases rapidly as the force on the wire is further increased. The wire elongates and breaks. The stress just before the wire breaks is called the breaking stress.

## ***Procedures:***

*Set-up of the apparatus*

1. The apparatus was set up on the bench top as shown below




- The wire was firmly clamped by using a G-clamp so that it does not slip.
- A white label sticker was fixed on to the copper wire to act as a marker such that it is about 50 cm from the pulley.
- A metre rule was fixed alongside the wire with the maker for measuring the extension.

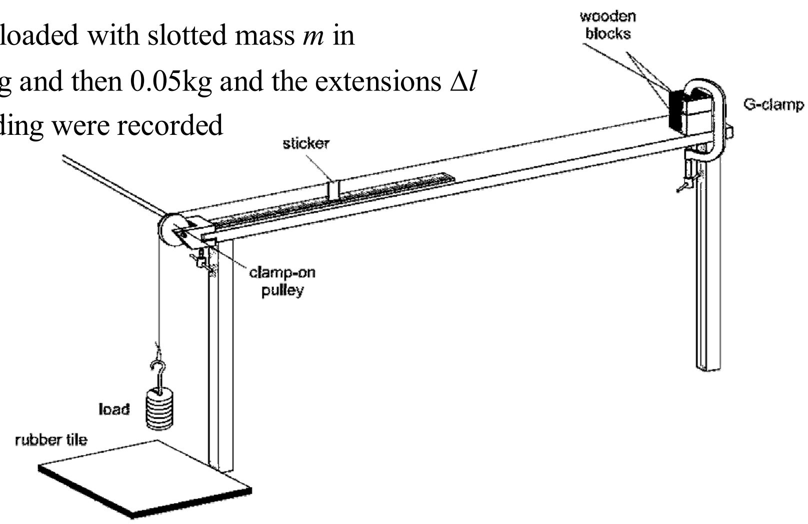
*Performance of the experiment*

1. The hanger was tied to the end of the wire so as to straighten out the kinks in the

wire and the unstretched length ($l_o$) of the wire from the edges of the wooden blocks up to the marker was measured.

2. A micrometer screw gauge was used to measure the diameter of the wire at different angles for each of the 8 location along the wire.

3. The wire was loaded with slotted mass $m$ in steps of 0.10kg and then 0.05kg and the extensions $\Delta l$ after each loading were recorded until the wire broke.




***Data table:***

Original length of wire $l_o = (3.000 \pm 0.001)$ m

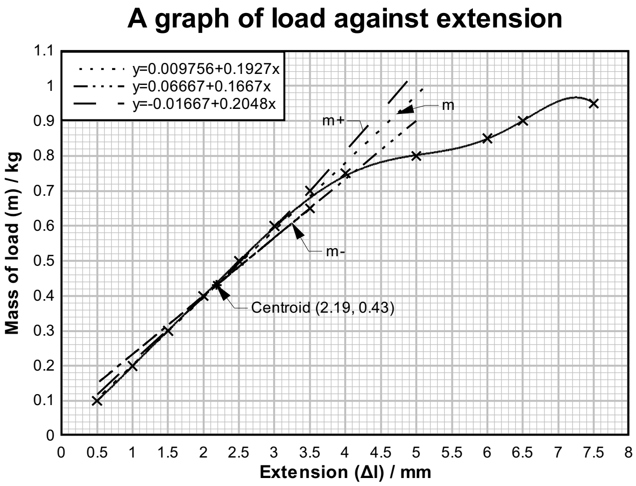
Percentage error in $l_o = \frac{0.001}{3.000} \times 100\ \% = 3.33 \times 10^{-2}\%$

| Diameter of the wire (mm) | | | |
|---|---|---|---|
| 0.255 | 0.250 | 0.225 | 0.230 |
| 0.225 | 0.255 | 0.225 | 0.255 |

Average diameter of the wire = (0.240 ± 0.005) mm

Percentage error in $d = \frac{0.005}{0.240} \times 100\ \% = 2.08\ \%$

| Readings for the graph: | | | | | | | |
|---|---|---|---|---|---|---|---|
| Load $m$/kg | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.65 |
| Extension $\Delta l$/mm | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 |
| Load $m$/kg | 0.70 | 0.75 | 0.8 | 0.85 | 0.90 | 0.95 | 1.00 |
| Extension $\Delta l$/mm | 3.5 | 4.0 | 5.0 | 6.0 | 6.5 | 7.5 | Broke |




## ***Data analysis:***

Young's modulus, $E = \dfrac{stress}{strain} = \dfrac{F/A}{\Delta l / l_o}$

where $F$ is the tension in the wire and $A$ is the cross-section area

Since $F = mg$ and $A = \dfrac{1}{4}\pi d^2$

$$E = \frac{\dfrac{mg}{\frac{1}{4}\pi d^2}}{\dfrac{\Delta l}{l_o}} = \frac{m}{\Delta l} \times \frac{4gl_o}{\pi d^2} = slope \text{ of the graph} \times \frac{4gl_o}{\pi d^2}$$

From the graph, the slope of the best fit line through the points of the straight line portion of the graph, $m = \dfrac{0.1927}{10^{-3}} = 192.7 \; kg \; m^{-1}$

$$E = 192.7 \times \frac{4gl_o}{\pi d^2} = 192.7 \times \frac{4 \times 9.8 \times 3.000}{\pi \times (0.24 \times 10^{-3})^2} = 125 \times 10^9 \; Pa$$

Assume that the cross-sectional area did not vary as the stress increased.

## ***Errors & accuracy:***

From the graph,

the slope of the best fit line: $m = \dfrac{0.1927}{10^{-3}} = 192.7 \; kg \; m^{-1}$

the maximum slope: $m^+ = \dfrac{0.2048}{10^{-3}} = 204.8 \; kg \; m^{-1}$

the minimum slope: $m^- = \dfrac{0.1667}{10^{-3}} = 166.7 \; kg \; m^{-1}$

Deviations: $m^+ - m = 12.1 \; kg \; m^{-1}$

Deviations: $m - m^- = 26.0 \; kg \; m^{-1}$

The maximum error in slope = larger of the deviations = $26.0 \; kg \; m^{-1}$

Slope of load-extension graph = $(192.7 \pm 26.0) \; kg \; m^{-1}$

Percentage error in slope:

$$\frac{26.0}{192.7} \times 100\% = 13.5\%$$

Percentage error in $E$

= % error in slope + % error in $l_o$ + 2 × % error in $d$

$= 13.5 + 0.033 + 2 \times 2.08$

$= 17.7\%$

Young's modulus of copper, $E = (125 \pm 22)$ GPa

## ***Conclusion:***

- The stress applied to a copper wire (s.w.g. 32) is directly proportional to the strain it produces before the extension becomes 3.5mm.
- The ratio of stress to strain will get smaller and not constant when the extension beyond 3.5mm (proportional limit), i.e. after the extension reached 3.5mm, small increase in stress can produce a great increase in strain.
- Copper obeys the Hooke's law.
- The Young's modulus of copper is $(125 \pm 22)$ GPa

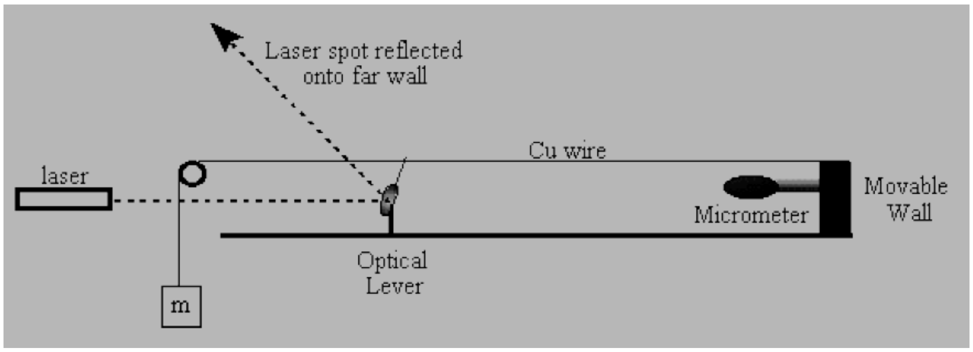
## ***Sources of Error:***

1. The copper wire did not have a constant cross-sectional area along its length.
2. There was fractional force due to the pulley applying to the wire.
3. Reading error in measuring the extension and the unstretched length.
4. Fluctuation of room temperature might change the diameter of the wire during the experiment.
5. The wire in the experimental set-up was not exactly horizontal that made our measurement of extension not accurate.
6. The cross sectional area of the wire got thinner under stress so that the expected stress would be less than the stress actually applied.

## ***Improvement of the Experiment:***

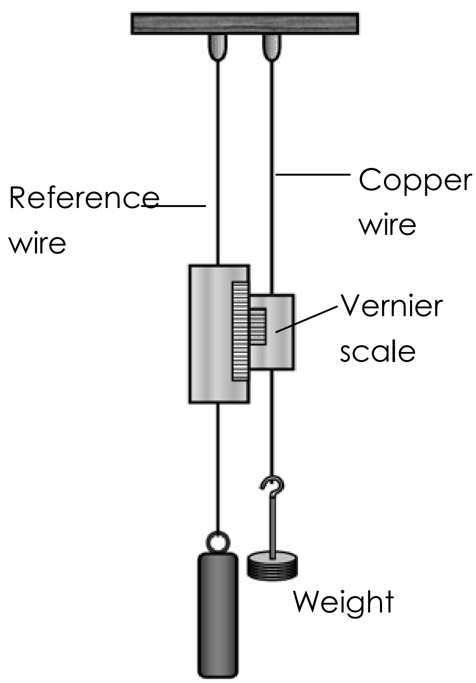
1. Fixed the metre ruler by another G-clamp so that measurement of the extension can be more accurate.

2. In order to measure extremely small extension with high precision, “optical lever” (a mirror mounted on a small pivot) can be used instead of just using a simple meter stick.




3. Repeat the experiment several time and take average of the extension values so that more accurate result can be obtained

4. The experiment can be repeated as below so that the small extension of the wire can be measured accurately by vernier scale; moreover, there will be no extra fractional fore due to the presence of pulley.




5. Repeat the experiment by using copper wire with different s.w.g and take an

average of the Young's modulus obtained so that we can estimate the value of Young's modulus of copper more accurately.

## ***Precautions:***

1. Wear safety goggle during the experiment so as to protect our eyes when the wire breaks eventually
2. The load should not be too high off the floor, and there should be a suitable soft landing platform, such as runner tile right below the load.
3. The unstretched length should be at least 3m for the wire to extend.

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