Charging and Discharging a Capacitor at Constant Rate

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Class: 6S
Class Number: 7

A. Objectives

- Learn how to calculate the charge stored in a capacitor or discharging it at a constant rate.
- Illustrate how to keep the charging and discharging currents constant.
- Show that the voltage across a capacitor is proportional to the change stored in it.
- Determine the capacitance of a capacitor directly from its definition.

B. Preview Questions

1. If a capacitor is charged at a constant rate, what do you say about the current through the capaciotor?

Ans: The current is constant.

2. If the capacitor discharges at a constant rate, what do you say about the current through the capacitor?

Ans: The current is constant.

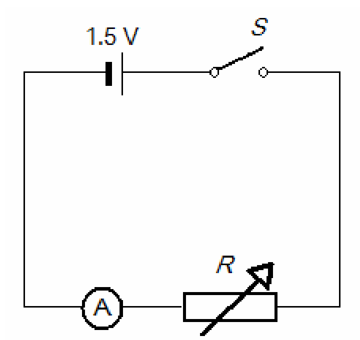
3. What is the charge stored in a capacitor if a steady current of 90 μA flows through it for 110 s?

Ans: Applying Q = It

$$Q = (110)(90 \times 10^{-6}) = 9900\ \mu C$$

4. If the terminals 1 and 2 of the variable resistor shown in Fig. 1 are used, how can you tell experimentally which direction the knob should be turned to give the highest resistance?

Ans: Connect p he circuit as shown. Set the variable resistor at the middle position initially. Slightly turn the variable resistor and observe how the meter deflects. The turning direction that gives a smaller current represents increasing r esistance.



C. Apparatus

- Variable resistor 0 – 50 kΩ 1
- Microammeter 1
- Digital voltmeter 1
- 2200μF capacitor 1
- Battery box with 2 dry cells 1
- Switch 1
- Stop watch 1

D. Theory

Charging capacitor at constant rate

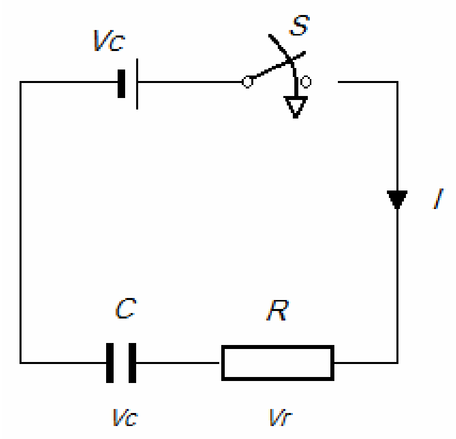


Fig.a Current falls with time

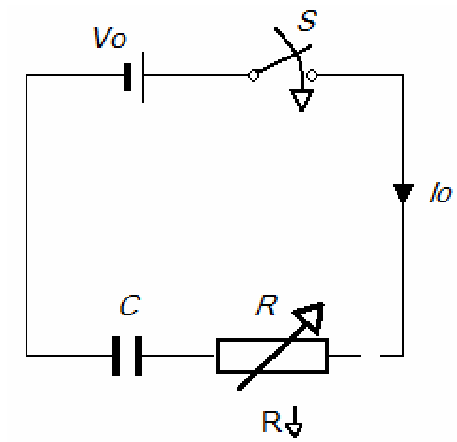


Fig.b Current can be kept constant

The capacitance *C* of a capacitor is defined as the charge *Q* stored in it per unit voltage. Thus, for a capacitor at a p.d. *V* with charge Q, the capacitance is $C = Q/V$.

In Fig.a, when the switch is closed, the current through the capacitor is the largest initially. As the capacitor is charged, V_c would increase, causing V_r to decrease. According to $V_r = IR$, *I* would decrease with time. In order to keep the current constant, you may reduce the value of R as shown in Fig. b. the constant current Io would flow for a while u ntil V_c reaches the e.m.f. if the cell, i.e. $V_c = V_r$. When the capacitor is fully charged, the current would fall abruptly to zero as shown in Fig. c. the corresponding variations in charge and voltage are shown in Fig.d and e respectively. This experiment has to be performed skillfully with patience, especially in the final stage where you may find it difficult to keep the current constant.

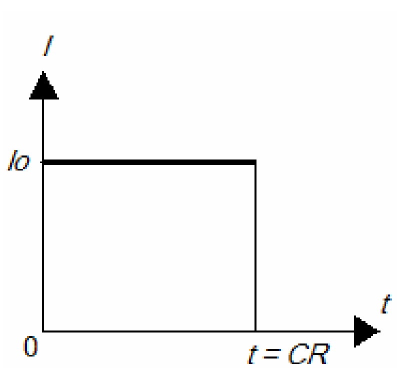


Fig.c Variation of current with time

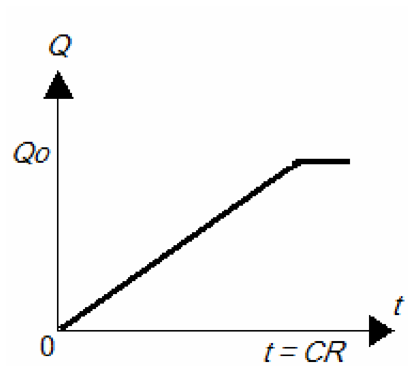


Fig.d Variation of charge with time

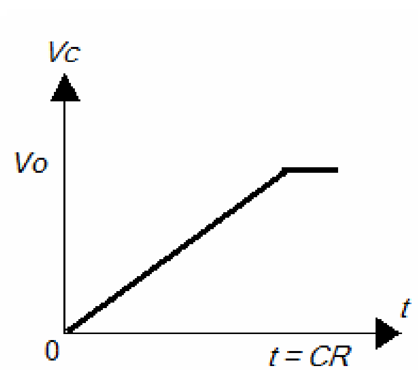


Fig.e Variation of voltage with

time

The final charges stored in the capacitor is given by $Q_o = I_o t$ and the final voltage across C is V_o. By $Q = CV$, the capacitance is given by $C = Q_o / V_o = I_o t / V_o$----- (1)

Discharging capacitor at constant rate

Fig.f shows the circuit for discharging the capacitor at constant rate. When the switch S is in position 1, the capacitor is fully charged. When it is changed to position 2, the capacitor would be discharged through R. Initially, R is set at high resistance. To keep the current constant, you will reduce the resistance so that the capacitor discharges at a constant rate.

The charge initially stored in C is calculated from the constant discharging current Io and the period of discharge t using $Q_o = I_o t$. Thus, the capacitance can also be found using the same equation 1 above.

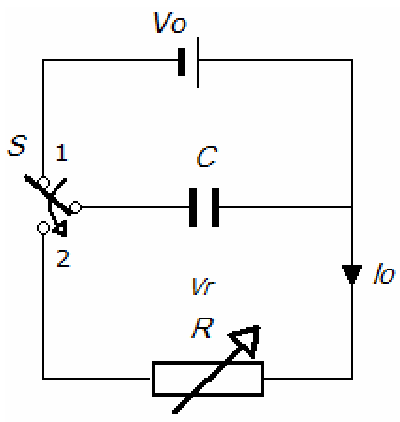


Fig.f Discharging t constant rate

The theoretical variation of current, charge and voltage with the same time for a discharging circuit at constant rate are shown in Fig.g to i below:

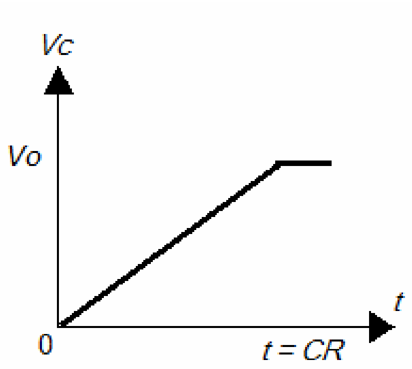


Fig.g Variation of current with time

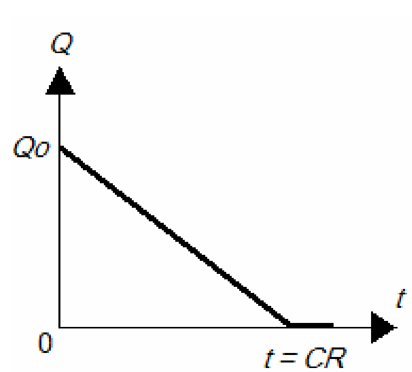


Fig.h variation of charge with time

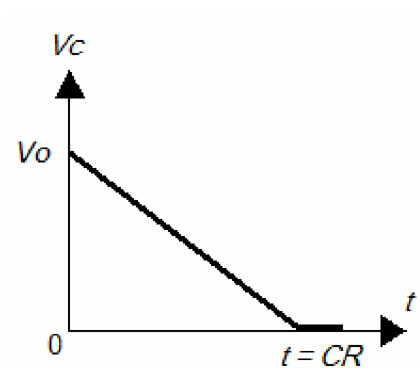
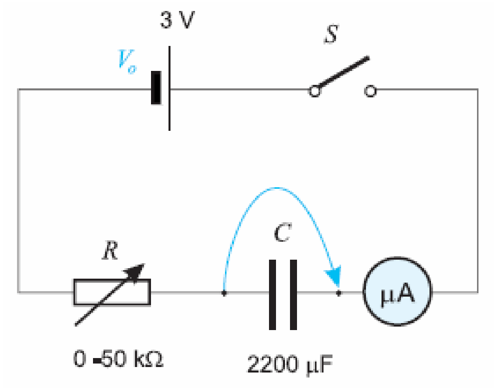


Fig.i Variation of voltage with time

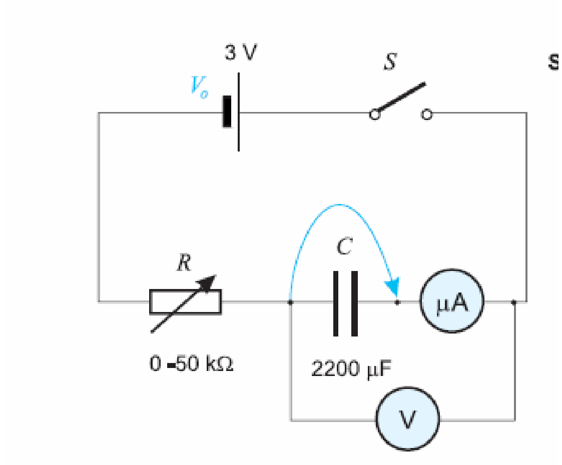
E. Procedures

Experiment 1 Charging a capacitor at a constant rate to find the total charge stored in the capacitor



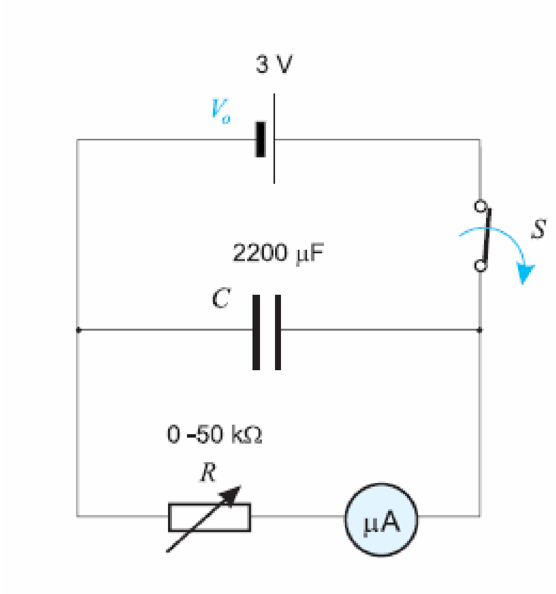
1. The circuit was examined.
2. The e.m.f. (V_o) of the battery was measured by the voltmeter. (Fig. 1)
3. The variable resistor was set to the highest resistance.
4. The circuit was connected is as shown above. The capacitor was discharged with a flying lead. The switch was closed and the initial charging current was recorded.
5. The capacitor was discharged again using the flying lead. (Fig. 2)
6. The flying lead was removed to charge the capacitor and timing was started immediately. The resistance was reduced to keep the charging current at its initial value. (Fig. 3)
7. The timer was stopped when the resistance of the variable resistor could not be reduced any more. The charging period was recorded.
8. Steps 5 to 7 were repeated twice.

Experiment 2 Variation of voltage with time when a capacitor is charged at constant rate



1. The above circuit was examined.
2. The circuit was connected as shown. The variable resistor was set to the highest resistance.
3. Switch S was closed. The capacitor was discharged with a flying lead. The initial ammeter reading I_o was written.
4. The flying lead was removed and timing was started immediately.
5. The resistance was reduced to keep the charging current at its initial value. The voltmeter reading V_c at each 10s intervals was recorded in Table 1.
6. Recording was stopped when the resistance of the variable resistor could not be reduced any more.

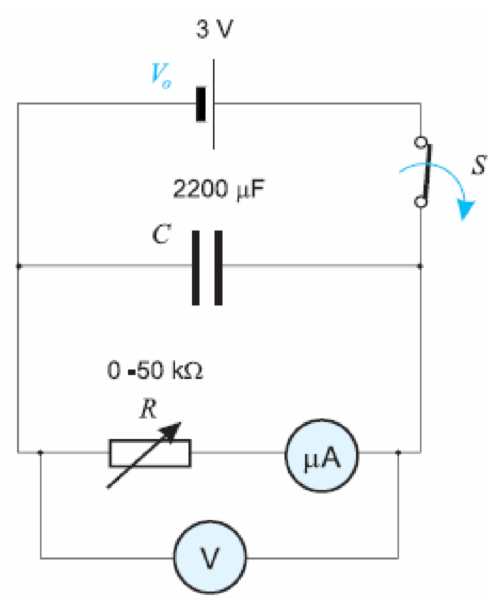
Experiment 3 Discharging a capacitor at a constant rate



1. The above circuit was examined.

2. The circuit was connected as shown. The variable resistor was set to the highest resistance.
3. Switch S was closed. The capacitor was fully charged very quickly. The ammeter reading I_o was recorded.
4. S was open and timing started immediately.
5. The resistance was reduced to keep the charging current constant at Io
6. The timer was stopped when the variable resistor could not be reduced any more. The discharging period was recorded.
7. Steps 3 to 6 were repeated twice.

Experiment 4 Variation of voltage with time when a capacitor is discharged at a constant rate



1. The above circuit was examined.
2. The circuit was connected as shown. The variable resistor was set to the highest resistance.
3. Switch S was closed, the ammeter reading I_o was recorded.
4. S was opened and timing started immediately.
5. The resistance was reduced to keep the charging current at Io. The voltmeter reading V_c at each 10s intervals was recorded in Table 2.
6. Recording was stopped when the resistance of the variable resistor could not be reduced any more.

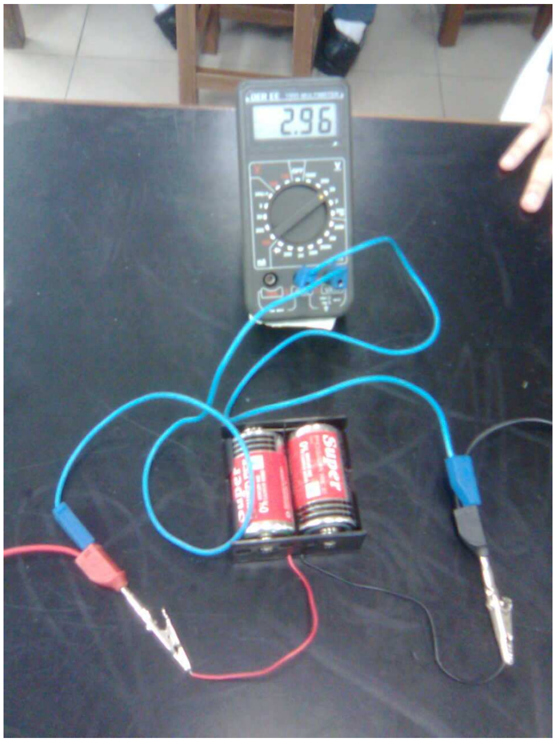


Fig. 1 Measuring the e.m.f. of the battery

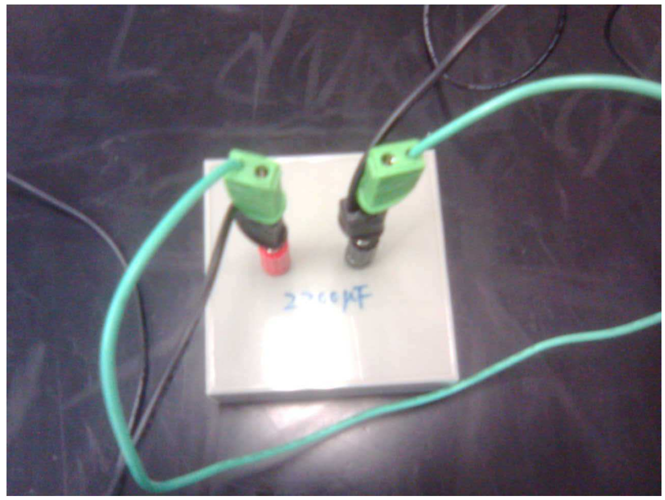


Fig. 2 Discharging the capacitor

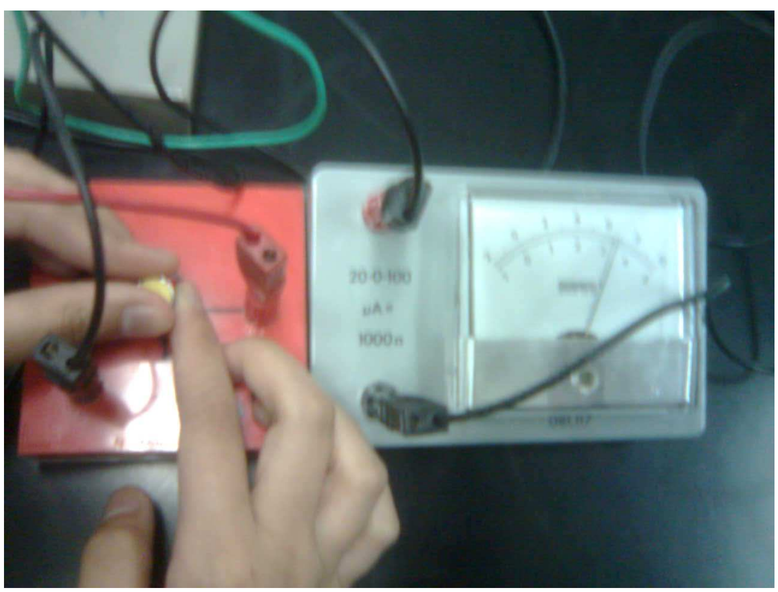


Fig.3 Adjusting the resistance of variable resistor to keep current constant

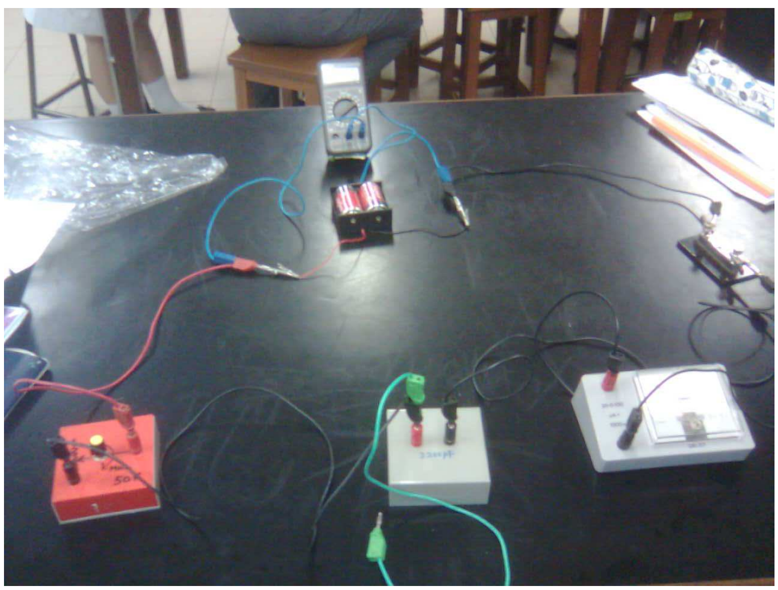
F. Precautions

- If you are not sure which direction the knob of the variable resistor that you should turn to give the highest resistance, you may either use a multimeter to test or connect up the circui t (using an ammeter) as shown. Note that you should set the variable resistor at the middle position initially. Slightly turn the variable resistor and observe how the meter deflects. The turning direction that gives a smaller current represents increasing resistance.
- Check the correctness of the polarities of the capacitor and the ammeter. Avoid causing the ammeter to deflect in the opposite direction.

G. Results, Calculations and Graphs

Experiment 1

Experimental set-up:



Step 2: e.m.f. of the battery, V_o = 2.96 ± 0.01V

Step 4: Initial charging current, I_o = 68 ± 2μA

Steps 7 – 9:

	Trial 1	Trial 2	Trial 3	Average charging period
Time recorded (s)	104	102	102	103 ± 0.01

Step 10: Total charge stored, Q = It

68 x 103 = 7004 μC

capacitance C, C = Q / V

= 7004 / 2.96

= 2366μF

Error:

By Q = CV and Q = It

C= It / V

$$\frac{\Delta C}{C} = \frac{\Delta I}{I} + \frac{\Delta t}{t} + \frac{\Delta V}{V}$$

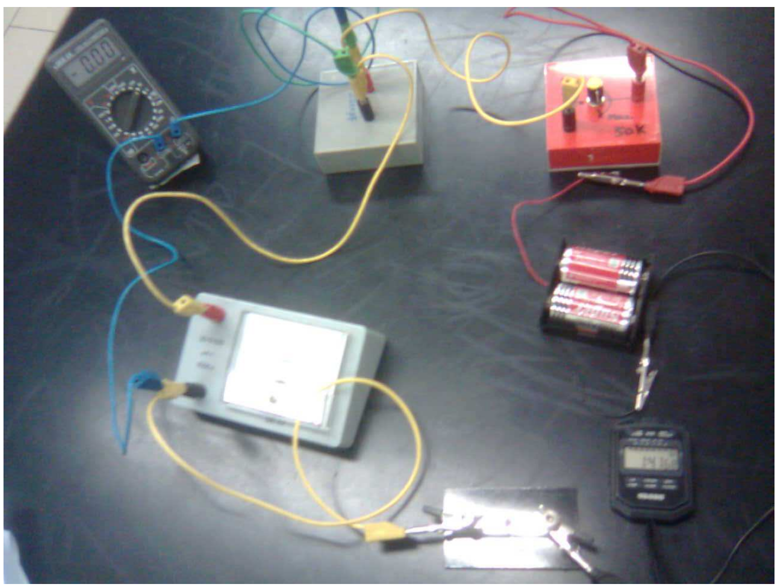
ΔC / C = (2/68) + (0.01/103) + (0.01/2.96)

= 3.29%

∴C = 2366 ± 78μF

Experiment 2

Experimental set-up:



Step 3: Initial ammeter reading, I_o = 68 ± 2μA

Table A

t (s)	10	20	30	40	50	60	70	80	90	100
V_c (V)	0.37	0.64	0.94	1.22	1.53	1.81	2.12	2.40	2.68	2.96
Q (μC)	680	1360	2040	2720	3400	4080	4760	5440	6120	6800

Data Analysis 2 Step 2

1. A straight line passing through the origin is obtained in the Q-V_c graph. Therefore, Q is proportional to V_c

2. $\because Q = CV_c$

   Slope of graph = $Q / V_c = C$
   Slope of graph = $C = [(5650 - 2250) \times 10^{-6}] / (2.5 - 1)$

C = 2267μF

Error:

From the graph, slope of the best fitted line = 2. 28 x 10^{-3}

Max. slope = $\frac{(5800 - 3500) \times 10^{-6}}{(2.5 - 1.55)}$ = 2.42 x 10^{-3}

Min. slope = $\frac{(3500 - 1200) \times 10^{-6}}{(1.55 - 0.5)}$ = 2.19 x 10^{-3}

Deviation: (2.42 - 2.27) x 10^{-3} = 0.15 x 10^{-3} (2.27 – 2.19) x 10^{-3} = 0.08 x 10^{-3}

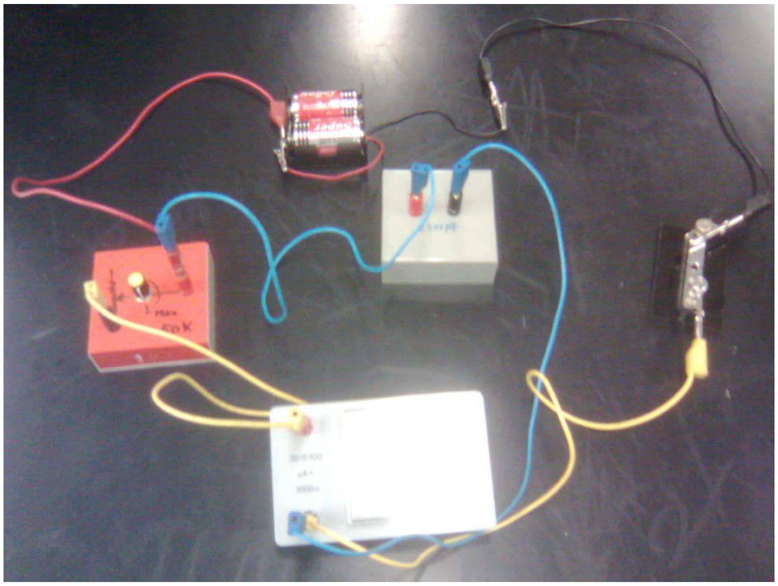
Maximum error in slope = larger deviation =

Percentage error in slope = (0.15 / 2.27) x 100% = 6.61%

∴C = 2267 ± 150μF

Experiment 3

Experimental set-up:



Step 3: Initial discharging current, I_o = 68 ± 2μA

Step 6 & 7:

	Trial 1	Trial 2	Trial 3	Average Discharging period
Time recorded (s)	100	99	99	99.3 ± 0.01

Step 8: Initial charge stored Q = It

68 x 99.3 = 6752μC

Capacitance, C = Q / V

= 6752/2.96

= 2281μF

Error:

By Q = CV and Q = It

$C = It / V$

$$\frac{\Delta C}{C} = \frac{\Delta I}{I} + \frac{\Delta t}{t} + \frac{\Delta V}{V}$$

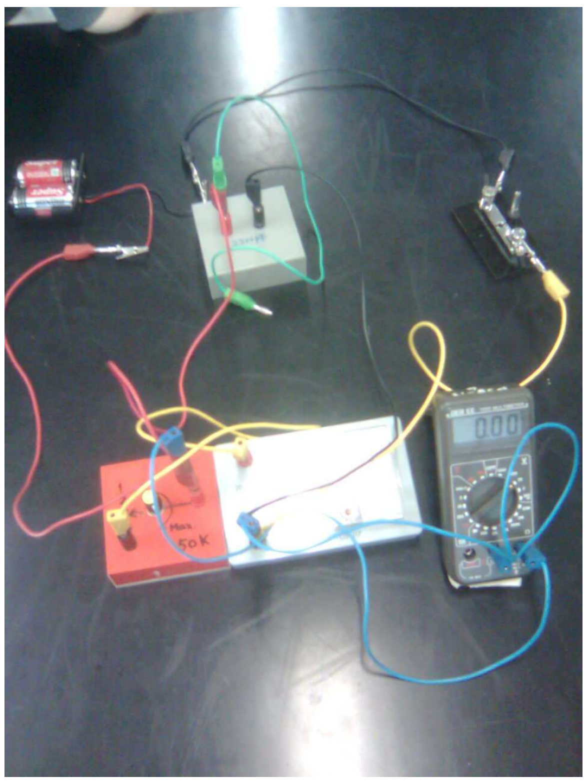
$\Delta C / C = (2/68) + (0.01/99) + (0.01/2.96)$

$= 3.29\%$

$\therefore C = 2281 \pm 75\mu F$

Experiment 4

Experimental set-up:



Step 3: Initial ammeter reading, $I_o = 68 \pm 2\ \mu A$

Table 2

t(s)	0	10	20	30	40	50	60	70	80	90	100
Vc(V)	2.96	2.63	2.34	2.04	1.74	1.43	1.14	0.85	0.56	0.26	0.00
Q(μC)	6374	6052	5372	4692	4012	3332	2652	1972	1292	612	0

Data Analysis 2 Step 2:

1. A straight line passes through origin is obtained in the Q -V_c graph. Therefore, Q is proportional to V_c

2. $\because Q = CV_c$

   Slope of graph = $Q/V_c = C$
   Slope of graph = $C = [(4550 – 1150) \times 10^{-6}] / (2 – 0.5)$

$C = 2267\ \mu F$

Error:

From the graph, slope of the best fitted line = 2.34×10^{-3}

Max. slope = $\dfrac{[(4500 - 2250) \times 10^{-6}]}{(1.95 - 1)}$ $= 2.37 \times 10^{-3}$

Min. slope = $\dfrac{[(5500 - 1250) \times 10^{-6}]}{(2.45 - 0.5)}$ $= 2.18 \times 10^{-3}$

Deviation: $(2.37 - 2.27) \times 10^{-3} = 0.10 \times 10^{-3}$ $(2.27 - 2.18) \times 10^{-3} = 0.09 \times 10^{-3}$

Maximum error in slope = larger deviation =

Percentage error in slope = (0.10 / 2. 27) x 100% = 4.41%

$\therefore C = 2267 \pm 100 \mu F$

H. Discussion

Answer to discussion

1. What are the sources of errors in the above experiments?

Ans:

Random error

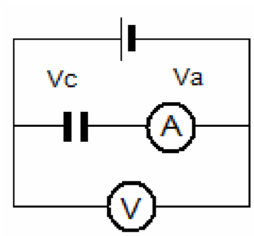
- Reading error of instruments e.g. microammeter, digital voltmeter and timer.

Systematic errors

- Zero error of microameter
- Resistance of wire
- Resistance of microammeter is not small enough
- Resistance of digital voltmeter is not infinity
- Poor experimental skills
  - fail to keep the current constant
  - poor reaction time

2. In experiment 2, the voltmeter is connected across both the capacitor an d the ammeter. Would it make any difference if it is connected across the capacitor only? State the pros and cons of the original arrangement.

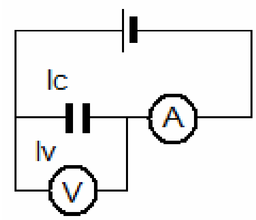
Ans:



When voltmeter are connected across both the capacitor and ammeter, the voltage measured does not indicate the actual voltage passing through the capacitor, but with the voltage of ammeter also as the ammeter is not ideal.

Therefore, $V = V_C + V_A$. As capacitor and ammeter is connected in series, ammeter reading is accurate.

When voltmeter is connected across the capacitor only, it is measuring the actual voltage passing through the capacitor. However, the ammeter is not measuring the actual current, but also the current passing through the voltmeter as it is not an ideal one.



The pro is the original arrangement is that the current is measured correctly, while the con is that the capacitance calculated will be smaller than the actual value.

3. The experiment uses the equation Q = It to determine the charge stored in a capacitor. Can we use an electrometer to measure the chare stored in the capacitor? Explain.

Ans:

Electrometer is adapted to measure charge with its build -in capacitor of capacitance 10^{-8}F, which can accept charges from its input terminal. It can measure charge only provided that the charged object has a low capacitance compare with the built-in capacitor C. otherwise, the charge in the capacitance cannot be fully transferred to the built -in capacitor C, therefore the actual no. of charge cannot be found out.

Assumptions

1. Zero error does not exist in mi croammeter
2. Microammter and digital voltmeter are ideal
3. Wires have no resistance

Limitations of apparatus

1. Zero error exists in microammeter
2. Resistance of wire
3. Resistance of microammeter is not small enough
4. Resistance of digital voltmeter is not infinity

Precautions

1. Take repeated readings and averaging to reduce systematic errors
2. Do not put the capacitor near any conductors to reduce stray capacitance
3. Select only one student for recording data

Suggestions for improvements

1. Use data logger to collect data
2. Use electronic device to keep the current constant

Comparison of result with that expected

From the experimental result, four values of capacitance C obtained are 2366 ± 78μF, 2267 ± 78μF, 2281 ± 75μF and 2267 ± 100μF respectively, while the actual value of the capacitance C is 2200μF.

The obtained values of 4 experiments are all higher than the expected value. This is because the there are errors existed during experiments , such as stray capacitance, reading errors, ammeter and voltmeter were not ideal.

I. Conclusion

By collecting data from 4 experiments, the average capacitance C is:

$$C = [(2366 \pm 78\mu F) + (2267 \pm 78\mu F) + (2281 \pm 75\mu F) + (2267 \pm 100\mu F)] / 4$$
$$= 2295\ \mu A$$

$$\text{Error} = (2295 - 2200) / 2200 \times 100\% = 4.33\%$$

From the experiment, we can find that the current of charging and discharging can be kept constant by adjusting the resistance of the variable resistance. Also, form experiment 2 & 4, straight lines passed through origins are obtained in the graphs of Q against V. therefore, the voltage across a capacitor is proportional to the charged stored in it.

Compare with the actual value of capacitance C, the experimental result C is higher than the actual one because of the presence of errors in the experiment.