Capacitance

Introduction

Chapter 3, “Capacitance,” contains laboratory experiments designed to explore the relationship between voltage and the amount of charge stored in an object. These experiments involve measuring electrical properties of capacitors in series, in parallel, while charging, discharging, and at varying widths between the surfaces. Hands-on experience and resulting data should provide insight into the nature of capacitance.

Theory

In order to charge an object, a certain amount of energy is required to transfer charge to that object. The energy per unit of charge is called voltage. Given a certain voltage, charge can be transferred to an object until the amount of energy that is required to add more charge exceeds the energy potential. A derived unit is useful for expressing the capacity of charge (in Coulombs) that can be transferred to an object per unit of voltage (in Volts). Therefore, a unit of capacitance called the Farad exists, and is defined as C = Q/V.

A capacitor comprised of two parallel surfaces will have a capacitance equal to

8.85 ρF/m, times the area of one of the plates, divided by the distance between them. When sharing the charge applied to one capacitor with a second capacitor, charge is conserved, therefore Vf * (C1 + C2) = Vi * C1. When discharging a capacitor through a resistor, V(t) = V0 * e-t/RC. When charging a capacitor through a resistor, V(t) = Vf – Vf * e-t/RC.

Experiments

3.5.2: Charging a Capacitor

This experiment required a 9V battery, a voltmeter, and voltage a follower that were assembled in this way: The battery and voltage follower ground contacts were connected to the volt meter ground, while the voltage follower output was connected to the positive terminal of the volt meter. To measure the voltage across the 0.033 μF capacitor, I connected one end of the capacitor to the positive lead from the voltage follower, and connected the other end to the ground.

To charge the capacitor, I touched the positive probe from the 9V battery to the ungrounded side of the capacitor. The voltmeter displayed 8.90 V after removing the battery probe. Therefore, the charge on the capacitor was 0.033 μF * 8.90 V = 0.294 μC.

3.6.1: Measuring Unknown Capacitance

This experiment was the same as the previous one, except that after the capacitor was charged, a second one was connected to it in parallel. When I connected the second capacitor, the measured voltage dropped to 6.87 V. So, theoretically, the capacitance of the second capacitor was C2 = C1Vi/Vf – C1 = 0.033 μF * 8.90 V / 6.87 V – 0.033 μF = 0.00975 μF. The second capacitor was actually rated at 0.01 μF, and the voltmeter was precise enough to measure the drop in voltage accurately within (0.00975 – 0.01) / 0.01 = -2.49%. The multi-meter or irregularities of the capacitors themselves could have caused such small error.

I repeated the experiment using one capacitor of unknown rating, and one of 0.033 μF. The voltage dropped this time to 8.24 V, which was unsatisfactory. Repeating again with a 0.0033 μF capacitor, the voltage read 5.43 V. This greater change in V indicated that the capacitances were of the same order of magnitude, allowing for greater accuracy in the following calculations. C1 = 0.0033 μF and Vf = 5.43 V. So, C2 = 0.0033 μF * 8.90 V / 5.43 V - 0.0033 μF = 0.00211 μF.

3.6.2: Variable-Gap Capacitor

This experiment measured the capacitance between two parallel plates that were connected directly to a capacitance meter. The distance between the plates was increased at prescribed intervals, and I recorded the capacitance for each interval. (See page 10 for these data.)

Data from the variable width capacitor experiment were graphed on log-log scales. The resulting scatter plot was very linear in shape. (See page 7.) To find the slope of this line, I divided the difference between the capacitance and distance coordinates after taking the log of each. I selected two points that fell exactly on the trend line, and used those points to find an equation for the line.

Point One = (log[2.4], log[13]) = (0.380, 1.114)

Point Two = (log[0.3], log[101]) = (-0.523, 2.004)

Slope = m = [(1.114 – 2.004) / (0.380 + 0.523)] = -0.9859

Using the general form of a line where the axes are labelled ‘x’ and ‘y’, ‘m’ is the slope, and ‘b’ is a constant equal to the y-intercept:

$y = (m)(x) + b$

$\log C = (m)(\log d) + b$

$\log C = (-0.9859)(\log d) + b$

Substituting the coordinates of Point Two where ‘C’ is the capacitance and ‘d’ is the distance between the plates:

$2.004 = (-0.9859)(-0.523) + b$

$b = 1.4888$

So, an equation for the trend line is:

$\log C = (-0.9859)(\log d) + 1.4888$

To compare this result with the current capacitance theory, I had to manipulate equation 3.2 so that it was in the form of $y = mx + b$.

$C = k\varepsilon 0A/d$

‘k’ is equal to one, ‘ε0’ equals 8.85 ρF/m, and ‘A’ is the surface area of one of the capacitor’s disks. For consistency with lab measurements, I converted all of the lengths to centimetres and all of the capacitances to picofarads. So, using ε0 = 8.85 * 10-2 pF/cm, and r = 10 cm;

$C = \varepsilon 0A/d$

$\log C = \log(\varepsilon 0A) – \log(d)$

$\log C = \log(8.85 * 10-2 * 102 * \pi) – \log(d)$

$\log C = (-1.0000)(\log d) + 1.4441$

Finally, this equation is comparable to my own, and illustrates a small margin of error.