What is Mathematics

By Clement Ng

What is mathematics? If you ask this question of the first person you meet on the street you will most likely hear that "Mathematics is the study of number." If you insist that your respondent be more specific, you may elicit the suggestion that mathematics is "The Science of number." But that is about as far as you will get, and it is not an adequate description of mathematics. It is out of date by 2500 years! The answer to the question "What is Mathematics?" has changed several since then.

Until around 500 BC, mathematics was indeed about numbers. Ancient Egyptian, Babylonian, and Chinese mathematics consisted almost solely of arithmetic. It was largely utilitarian and very much of a "cookbook" variety. ("Do such and such to a number and you will get the answer.")

Between 500BC and AD300, Mathematics expanded beyond the study of number. The mathematicians of ancient Greece were concerned more with geometry. Indeed, they regarded numbers in a geometric fashion, as measurements of length, and when they discovered that there were lengths to which their numbers did not correspond (called irrational lengths), their study of number largely came to halt. For the Greeks, with their emphasis on geometry, mathematics was about number and shape.

Only with Greeks did mathematics change from a collection of techniques for measuring, counting, and accounting into an academic discipline having both aesthetic and religious elements. At the start of the Greek period, Thales introduced the idea that precisely stated assertions of mathematics could be logically proved by formal argument. For the Greeks, this approach culminated in the publication, around 350BC, of Euclid's mammoth thirteen volume text Elements, reputedly the second most widely circulated book of all time after the Bible.

After the Greeks, although mathematics advanced in several parts of the world – notably in Arabia and China – its nature did not change until the middle of the seventeenth century, when Sir Isaac Newton (in England) and Gottfried Wilhelm Leibnitz (in Germany) independently invented the calculus. In essence, the calculus is the study of motion and change. Before calculus, mathematics had been largely restricted to the static issues of counting, measuring and describing shape. The new techniques to handle motion and change enabled mathematicians to study the motion of the planets and of falling bodies on the earth, the workings of machinery, the flow of liquids, the expansion of gases, physical forces such as magnetism and electricity, flight, the growth of plants and animals, the spread of epidemics, and the fluctuation of profits. Mathematics became the study of number, shape, motion, change, and space.

At first, calculus was mainly directed toward the study of physics, and many of the great seventeenth and eighteenth century mathematicians were also physicist. But from about 1750 onward there was an increasing interest in the mathematical theory, not just its applications, as mathematicians sought to understand what lay behind the enormous power of calculus. By the end of the nineteenth century, mathematics had become the study of number, shape, motion, change, space, and of the mathematical tools that are used in this study. This was the start of modern mathematics.

The growth of mathematical activity in the present century can best be described as an explosion in knowledge. In 1900, all the world's mathematical knowledge would have fit into eighty books. Today it would take maybe 100,000 volumes to contain all known mathematics. Not only have existing branches, such as geometry, calculus continued to grow, but many quite new branches have sprung up. At the turn of the century, mathematics consisted of about twelve subjects: arithmetic, geometry, calculus, and so on. Today, there are between sixty and seventy distinct categories. Some subjects, like algebra or topology, have split into subfields; others, such as complexity theory or dynamical systems theory, are completely new.

Given such diversity, how does today's mathematician answer the question, "What is mathematics?" The most common answer is that mathematics is the science of patterns. Much of the impact of the phrase "the science of patterns" comes from its brevity, But brevity comes at a price of possible misunderstanding. In this case, the word "patterns" requires some elaboration. It certainly is not restricted to visual patterns such as wallpaper patterns or the pattern of tiles on the bathroom floor, although both can be studies mathematically. A slightly fuller definition would be: Mathematics is the science of order, patterns, structure, and logical relationships. But since the meaning mathematicians attach to the word "pattern" in this context includes all the terms in the expanded definition, the shorter version says it all – provided you understand what is meant by "pattern".

The patterns studied by the mathematician can be either real or imagined, visual or mental, static or dynamic, qualitative or quantitative, utilitarian or recreational. They arise from the world around us, from the depths of space and time, and from the workings of the human mind. Different kinds of patterns give rise to different branches of mathematics. For example, number theory studies (and

arithmetic uses) the patterns of number and counting; geometry studies pattern of reasoning; probability theory deals with patterns of chance, topology studies patterns of closeness and position.
The patterns and relationships studied by mathematicians occur everywhere in nature: the symmetrical patterns of flowers, the often complicated patterns of knots, the orbits swept out by planets as they move through the heavens, the pattern of spots on a leopard's skin, the voting patterns of a population, the pattern produced by the random outcomes in a game of dice or roulette, the relationship between the words that make up a sentence, the patterns of sound that we recognise as music. Sometimes the patterns are numerical and can be described using arithmetic – voting patterns, for example. But often they are numerical – for example, patterns of knots and symmetry patterns of flowers have little to do with numbers.
Because it studies such abstract patterns, mathematics often allows us to see – and hence perhaps make use of – similarities between two phenomena that at first appear quite different. Thus we can think of mathematics as a pair of conceptual spectacles that enables us to see what would otherwise be invisible – a mental equivalent of the physician's X ray machine or the soldier's night vision goggles. With mathematics, we can make the invisible visible.
Without mathematics, there is no way you can understand what keeps jumbo jet in the air. As we all know, large metal objects don't remain off the ground without something to support them. But when you look at a jet aircraft flying overhead, you can't see anything holding it up. It makes mathematics – in this case an equation discovered by the mathematician Daniel Bernoulli early in the eighteenth century – to "see" what keeps an airplane aloft.
What is it that causes objects other than aircraft to fall to the ground when we release them? "Gravity," you answer. But that's just giving it a name. It's still invisible. We might as well call it magic. Newton's equations of motion and mechanics in the seventeenth century enables us to "see" the invisible forces that keep the earth rotating around the sun and cause an apple to fall from the tree onto the ground.
Both Bernoulli's equation and Newton's equations use calculus. Calculus works by making visible the infinitesimally small. That's another example of making the invisible visible.
Here's another. Two thousand years before we could send spacecraft into outer space to photograph our planet, the Greek mathematician Erastothenes used mathematics to show that the earth was round. Indeed, he calculated its diameter, and hence its curvature, with considerable accuracy.
Physicists are using mathematics to try to see the eventual fate of universe. In this case, the invisible that mathematics makes visible is the invisible of the not yet happened. They have already used mathematics to see into the distant past, making visible the otherwise invisible moments when the universe was first created in what we call the big bang.
Coming back to earth at the present time, how do you "see" what makes pictures and sound of a football fame so miraculously appear on a television screen on the other side of town? One answer is that the pictures and sound are transmitted by radio waves – a kind of electromagnetic radiation. But as with gravity, that just gives the phenomenon a name, it doesn't help us to "see" it. You need Maxwells' equation, discovered in the last century, to "see" the otherwise invisible radio waves.
Mathematics is not some game that people make up, but it is about the patterns that arise in the world around us. Mathematics is not about numbers, but about life. It is about the world in which we live. It is about ideas. And far from being dull and sterile, as is so often portrayed, it is full of creativity. Many people have compared mathematics to music. Certainly the two have a lot in common, including the use of an abstract notation. Indeed, the first thing that strikes anyone who opens a typical book of mathematics is that it is full of symbols – page after page of what looks like a foreign language written n a strange alphabet. In fact, that's exactly what it is. Mathematicians express their ideas in the language of mathematics.

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