

## Maths Coursework - Artic Research

Speed of wind - 50 km/h blowing from the west

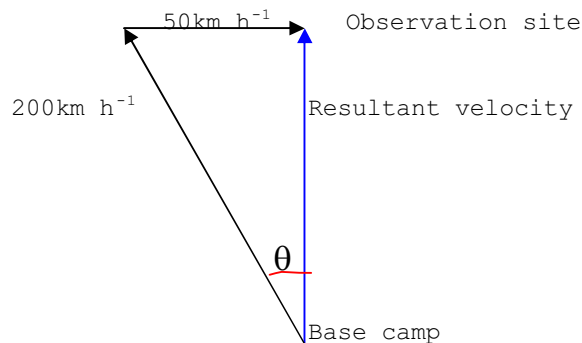
Speed of aeroplane - 200 km/h

I will treat the aircraft as a particle, instant and constant speed will be assumed, take off and landing times will be ignored.

The aircrafts speed will be affected by the speed and direction on the wind. As we assume the aircraft speed, wind speed and direction will be constant; we can use vector diagrams to work out how long the journey will take.

I will have my first base camp in the middle. I will then place the base camp at different areas in the circle, and see how this effects the journey time. I will then work out a general formula for any speed of aircraft, wind speed and direction, so the journey time could be worked for any variables of these 3.

Circle 1 - radius = 50km. First observation site - north ( $0^\circ$ ). Resultant velocity will have to act from the base camp to the observation site.



$$\text{Angle } \theta = \text{Sine } \theta = 50 / 200$$

$$\theta = 1/4$$

$$\theta = \text{Sine}^{-1} (1/4)$$

$$\theta = 11.25^\circ$$

This angle  $11.25^\circ$  can be used to work out the bearing of the aircraft. Bearing =  $360^\circ - 11.25^\circ$

$$= 348.75^\circ$$

So how long would the journey take? First we have to use Pythagoras' theorem to work out the speed of the aircraft. Pythagoras' theorem is "the sum of the squares of the opposite and adjacent = hypotenuse"

$$\text{or } a^2 + b^2 = x^2$$

$$200^2 = v^2 + 50^2$$

$$200^2 - 50^2 = v^2$$

$$37500 = v^2$$

$$v = \sqrt{37500}$$

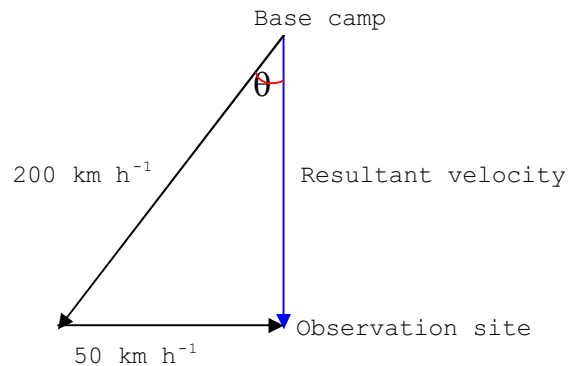
$$\text{Resultant speed of aircraft} = 193.6 \text{ km h}^{-1} \text{ (3s.f)}$$

We can now work out the time using the speed, distance, and time formula.

$V = S/T$  (speed = distance/time)  
 $T = S/V$   
 $T = 50/193.6$   
 $T = 0.258$  hours (3s.f)  
 Time = 15 minutes 48 seconds

This is the time just going to the base camp. The vector diagram coming back will look exactly the same for this degree, so the total time is  $2T = 2 \times 0.258 = 0.516 = 31$  minutes 36 seconds.

The vector diagram for the observation site south ( $180^\circ$ ) will mirror the one going north.



$$\theta = \text{Sine}^{-1} (1/4)$$

$$\theta = 11.25^\circ$$

$$\text{Bearing} = 360^\circ - 11.25^\circ$$

$$\text{Bearing} = 348.75^\circ$$

Speed of aircraft = resultant velocity

$$200^2 = v^2 + 50^2$$

$$200^2 - 50^2 = v^2$$

$$37500 = v^2$$

$$v = \sqrt{37500}$$

$$\text{Resultant speed of aircraft} = 193.6 \text{ km h}^{-1} \text{ (3s.f)}$$

$V = S/T$  (speed = distance/time)

$$T = S/V$$

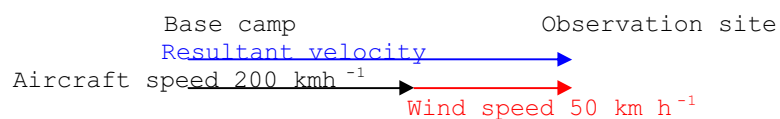
$$T = 50/193.6$$

$$T = 0.258 \text{ hours (3s.f)}$$

$$\text{Time} = 15 \text{ minutes } 28 \text{ seconds}$$

As with the time going north, the total time will be  $2T$ , as the vector diagram will be the same in both directions. Total time is  $2T = 2 \times 0.258 = 0.516 = 31$  minutes 36 seconds.

The next observation site will be east of the base camp, at  $90^\circ$ .



$$\text{Speed of aircraft} = 200 \text{ kmh}^{-1} + 50 \text{ kmh}^{-1}$$

$$= 250 \text{ kmh}^{-1}$$

Time taken to reach observation site -  $T = S/V$

$$T = 50 / 250$$

$$T = 0.2 \text{ hours}$$

$$T = 12 \text{ minutes}$$

Resultant velocity returning from observation site to base camp -

Base camp

Observation site

Resultant velocity 

Aircraft speed  $200 \text{ kmh}^{-1}$  

Wind speed  $50 \text{ kmh}^{-1}$  

$$\text{Aircraft speed} = \text{resultant velocity} = 200 - 50 = 150 \text{ kmh}^{-1}$$

Time taken to return to base camp -  $T = S/V$

$$T = 50 / 150$$

$$T = 0.333 \text{ hours}$$

$$T = 20 \text{ minutes}$$

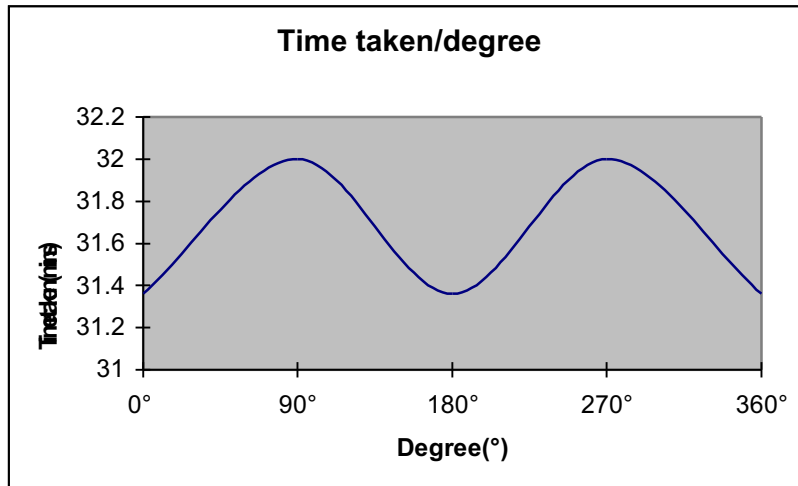
Total time taken to travel to observation site and return = 20 minutes + 12 minutes = 32 minutes

The time taken to reach the observation site west ( $270^\circ$ ) of the base camp will be the same as the time taken to reach the observation site east of the base camp. This is because the vector diagram travelling from the base camp to the east observation site will be the same the vector diagram from the west observation site to the base camp, and the vector diagram from the base camp site to the east observation site will be the same as the vector diagram travelling from the west observation site to the base camp. So total time taken to reach west observation site and return to base camp = 32 minutes.

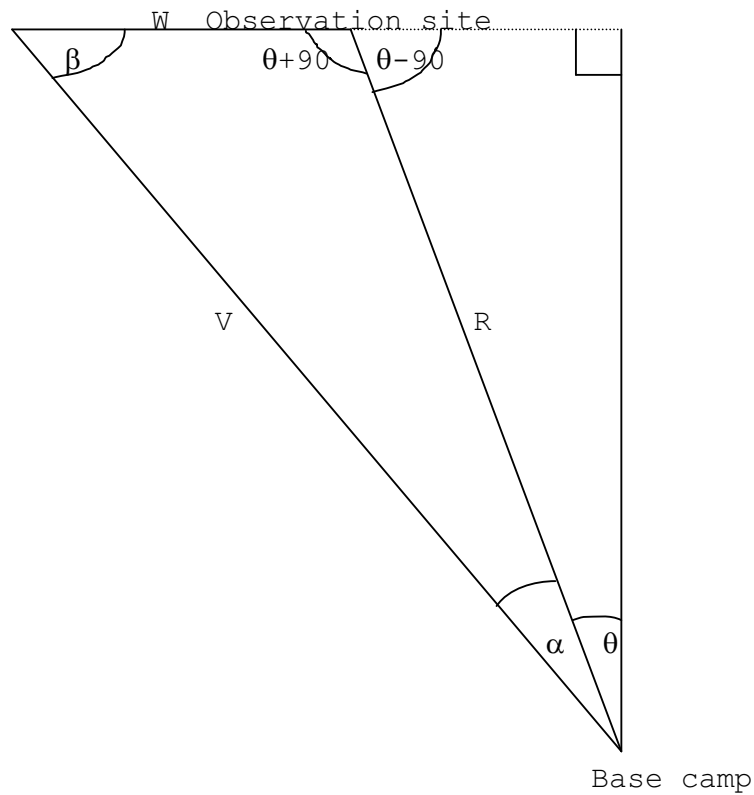
The results taken would not be very realistic, as the journey times are quite short take off and landing times will be significant. In a place like the arctic take off and landing times would be very significant, as it would take variable and mostly long periods of time to get the aircraft prepared. The times would be variable because the weather is so unpredictable in the Arctic. The journey may only be, say 32 minutes, but the preparation time could be much longer. For a model of this simplicity, we don't take into account this extra time for take offs and landings; we just treat the aircraft as a particle, assuming instantaneous and constant speed.

The speed of the wind is also unlikely to be constant in direction and speed.

It is clear from the readings taken that a graph can be drawn that repeats itself after  $180^\circ$ . The time taken for  $0^\circ$  is the same as the time taken for  $180^\circ$ , and the total time for the site  $90^\circ$  is the same as the degree  $180^\circ$  later,  $270^\circ$ .



I now need to derive a formula to calculate the time taken for any wind speed, aircraft speed and wind direction.



R = resultant velocity  
W = wind speed  
V = aircraft speed

R = ?

Using sin rule  
 $\sin A / a = \sin B / b$

$$\sin (90+\theta) / V = \sin \alpha / W$$

$$\text{Therefore, } \sin^{-1} (W \sin 90 + \theta / V) = \alpha$$

$$\sin 90 + \theta = \cos \theta$$

$$\text{Therefore, } \sin^{-1} (W \cos \theta / V) = \alpha$$

$$\text{There are } 180^\circ \text{ in a triangle, therefore } 180 = (90 + \theta) + \alpha + \beta$$

$$\text{So } 90 - \theta - \alpha = \beta$$

$$\text{As } \alpha = \sin^{-1} (W \cos \theta / V), \text{ we can substitute this for } \alpha \text{ in } 90 - \theta - \alpha = \beta$$

$$\text{Therefore, } 90 - \theta - [\sin^{-1} (W \cos \theta / V)] = \beta$$

We can then substitute this back into the sin rule

$$\sin \beta / R = \sin (90 + \theta) / V$$

We can use this to work out the value of R, by making it the subject of the formula

$$V \sin \beta / \cos \theta = R$$

Therefore,

$$V \sin [90 - \theta - \sin^{-1} (w \cos \theta / V)] \text{ all divided by } \cos \theta = R$$

We can then use this to work out the time it will take to make the journey, using  $v = s/t$  -  $t = s/v$

Where  $s$  = radius of circle =  $R$ , and  $v$  = the resultant velocity

$$\mathbf{R / V \sin [[90 - \theta - \sin^{-1} (w \cos \theta / V)] / \cos \theta] = t}$$

This formula will give the time taken to travel to the observation site.

To calculate the time taken to return to base camp,  $180^\circ$  needs to be added to  $\theta$ . This is because the bearing at  $\theta + 180^\circ$  = the bearing for the vector going in the opposite direction to the original vector, which equals the return journey time.

$$\mathbf{R / V \sin [[90 - (\theta + 180) - \sin^{-1} (w \cos (\theta + 180) / V)] / \cos (\theta + 180)] = t}$$

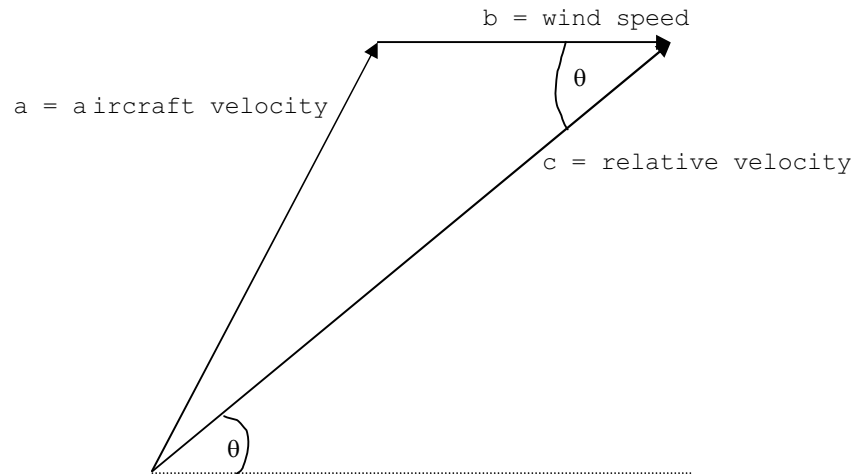
These formulas will give me the time taken for any aircraft speed, and wind speed or direction, where -

$S$  = radius of circle

$V$  = velocity of the aircraft

$W$  = wind speed

We can derive another formula to calculate the speed of the resultant vector using the quadratic equation. This will only give the speed from the centre of the circle.



Both the plane and the wind have constant speed.

$$a^2 = b^2 + c^2 - 2bc \cos \theta$$

$$\text{Therefore } c^2 + (-2b \cos \theta) c + (b^2 - a^2) = 0$$

Using the quadratic formula, we can take this as a quadratic in  $C^2 - C = 2b \cos \theta = \sqrt{(-2b \cos \theta)^2 - 4(b^2 - a^2)}$

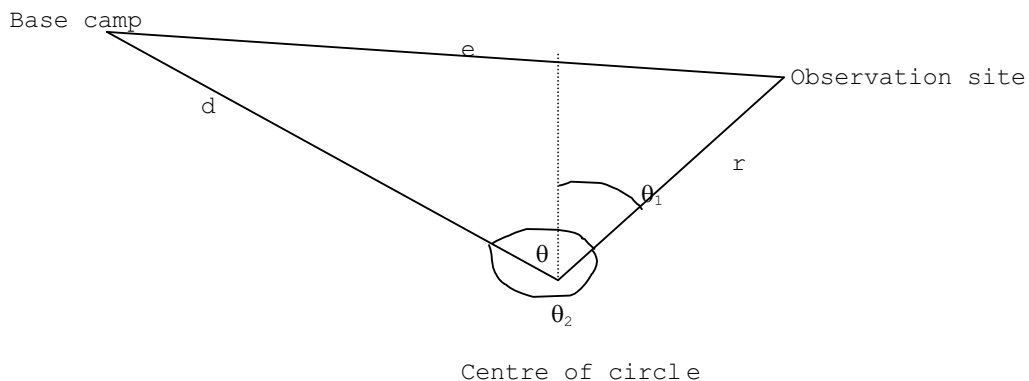
$$\text{Therefore, } C = 2b \cos \theta \pm \sqrt{4b^2 \cos^2 \theta - 4(b^2 - a^2)}$$

Where  $b = \text{wind speed}$ ,  $a = \text{aircraft speed}$ ,  $c = \text{resultant velocity}$ ,  $\theta = \text{angle of resultant relative to the wind}$ .

To work out the distance the plane needs to travel from any base to any point on the circumference of the circle, we need to know where the base and the observation site is.

Observation site: measured in bearing from the centre of the circle, and will be the radius away from the centre of the circle.

Base camp: measured in bearing from the centre of the circle, with the distance in km.



Where  $d$  = distance,  $e$  = distance to observation site from base camp,  
 $r$  = distance to observation from centre of circle (the radius),  $\theta$  =  
 observation site bearing minus base bearing ( $\theta = \theta_2 - \theta_1$ ).

Using cosine rule -  $a^2 = b^2 + c^2 - 2bc \cos A$ , where  $a = e$ ,  $b = d$ ,  $c = r$ ,  $A = \theta$ .

$$\text{Therefore, } e^2 = d^2 + r^2 - 2 \times r \times d \cos \theta$$

This formula can be used to calculate the length of the resultant velocity from any given point.

**Conclusion** - the formula to find time taken to travel to the observation site:

$$R / V \sin [ [90 - \theta - \sin^{-1} (w \cos \theta / V)] / \cos \theta ]$$

To calculate the time taken to return to base camp;

$$R / V \sin [ [90 - (\theta + 180) - \sin^{-1} (w \cos (\theta + 180) / V)] / \cos (\theta + 180) ]$$

Where  $R$  = radius of the circle,  $V$  = velocity of the aircraft,  $W$  = wind speed.

So total time to and from base camp:

$$[R / V \sin [ [90 - \theta - \sin^{-1} (w \cos \theta / V)] / \cos \theta ]] + [R / V \sin [ [90 - (\theta + 180) - \sin^{-1} (w \cos (\theta + 180) / V)] / \cos (\theta + 180) ]]$$

Formula to calculate the speed of the resultant vector;

$$C = 2b \cos \theta \pm \sqrt{4b^2 \cos^2 \theta - 4(b^2 + a^2)}$$

Where  $b$  = wind speed,  $a$  = aircraft speed,  $C$  = resultant velocity,  $\theta$  = angle of resultant relative to the wind.

Formula to calculate the length of the resultant velocity from any given point;

$$e^2 = d^2 + r^2 - 2 \times r \times d \cos \theta$$

Where  $d$  = distance from centre of circle to observation site,  $e$  = distance to observation site from base camp,  $r$  = distance to observation from centre of circle (the radius),  $\theta$  = observation site bearing minus base bearing ( $\theta = \theta_2 - \theta_1$ ).