Coursework Title: Lines

Candidate name: Shoaib Muhammad

Candidate number: 5115

Centre name: Geoffrey Chaucer Technology College

Centre number: 10818

Plan

Firstly In this coursework I will draw six or more lines which will cross each other and while doing this I hope to get as much crossover points as I can, as well as I will try to get the maximum regions. I will try to avoid any sort of double intersecting i.e. intersecting over a ready made crossover point.

I will try and keep all my lines as consistent (placement). To make sure I do this I will draw 1 line and then copy past it and then add another line this will help me keep my lines the same i.e. consistent. But I will vary the number of lines I use. I will draw six line models (each having one more line than the previous one) the first model that I will create will have only one line the second will have two lines the third will have three lines etc.

Secondly I will try to figure out the formulas that will find the n^{th} term for the crossover points, open regions, closed regions and the total regions. I also plan to find the relationships / sequence between any two of these characteristics of the line(s). I will accomplish this by using the following formula: A+B (n+1) +0.5(n-1) (n-2) C.

A = 1^{st} term of the sequence.

B = the difference between the first two terms.

C = the 2^{nd} difference (the difference of the 1^{st} difference).

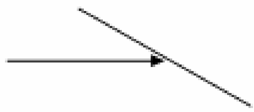
Thirdly I will put all my findings onto to a table consisting of total regions, closed regions, open regions, crossover points and all of the formulas.

Lastly I will try to summarize all my findings by writing up a conclusion and evaluation.

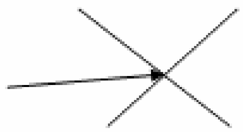
Prediction

I am going to predict that every time the number of lines increases by one the number of open regions will increase by two. For e.g. if there are two lines then the number of open regions will be four. In other words the number of open regions will double the number of closed lines. I predict that the relationship between open regions and closed regions will be a sequence that has a 1^{st} difference and a 2^{nd} difference. This will have to be the case, if I am to work out the nth term in the way I have described above. I also predict that there will always be a greater number of closed regions in comparison to open regions and finally that the number of total regions will be greater than the number of crossover points.

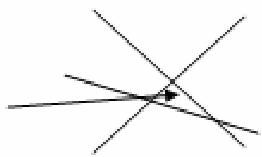
Diagrams showing my investigation



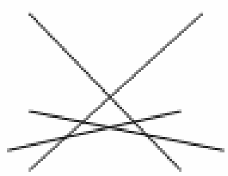
0 Crossover points
2 open regions
0 closed regions
Total regions: 2



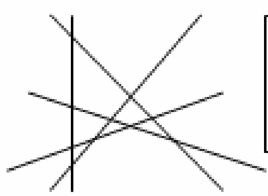
1 Crossover points
4 open regions
0 closed regions
Total regions: 4



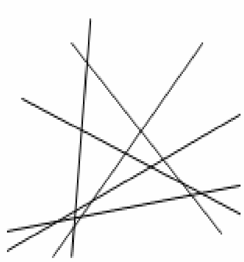
3 crossover points
6 open regions
1 closed regions
Total regions: 7



6 crossover points
8 open regions
3 closed regions
Total regions: 11



10 crossover points
10 open regions
6 closed regions
Total regions: 16



16 crossover points
12 open regions
10 closed regions
Total regions: 22

Summarizing of the 6 lines that I drew with the formulas

Shoaib Muhammad							
							Formulas
No. of lines	**1**	**2**	**3**	**4**	**5**	**6**	
Crossover points	0	1	3	6	10	15	(0.5n2)–(0.5n)
Open regions	2	4	6	8	10	12	2n
Closed regions	0	0	1	3	6	10	$(0.5n^2)-(1.5n)+1$
Total regions	2	4	7	11	16	22	(0.5n2)+(0.5n) +1

Patterns I managed to spot while investigating

While undergoing the investigation I found numerous amount of patterns e.g. the relationships between open & closed regions. The following information will give you insight information about what I managed to find i.e. the relationship between open and closed regions.

Table shows relationship between open and closed regions

No. of Lines	1	2	3	4	5	6
Open Regions	2	4	6	8	10	12
Closed Regions	0	0	1	3	6	10
Difference	2	4	5	5	4	2

From the above table I can gather that the number of closed regions for any particular number of lines is less than the number of open regions for the same number of lines. The number of open regions is double than the number of line. But there isn't any particular relationship between the number of lines and the number of closed regions which can be gathered from this table. However, there is an extra row which shows the difference between the open region and the closed regions for any particular number of lines and this shows a pattern. The difference goes up for the first three lines, where it peaks. Following on from that it remains constant for one more line after which it starts to decline in the same pattern as it increased.

Table shows relationship between open and total regions

No. of Lines	1	2	3	4	5	6
Open Regions	2	4	6	8	10	12

Total Regions	2	4	7	11	16	22
Difference	0	0	-1	-3	-6	-10

Table shows relationship between closed total regions

No. of Lines	1	2	3	4	5	6
Closed Regions	0	0	1	3	6	10
Total Regions	2	4	7	11	16	22
Difference	-2	-4	-6	-8	-10	-12

The n^{th} term

The n^{th} term is useful in order to predict the next number in any particular sequence without actually going through the trouble of working it out, which in this case means drawing the lines and counting the number of open regions for example. It can be worked out by using the formula below:
A+B (n-1) +0.5(n-1) (n-2) C
Where,
A = 1st term in the sequences below.
B = The 1st difference between the two different terms in the sequences below.
C = the 2nd difference in the sequences below which is always constant.

Working out the n^{th} term for closed regions

Sequence	0 0 1 3 6 10
1st Difference	0 1 2 3 4
2nd Difference	1 1 1 1

Consequently,
A = 0
B = 0
C = 1
The equation for closed regions is = 0+0(n+1) +0.5(n-1) (n-2)1
Simplified = $(0.5n^{2)}$-(1.5n) +1
Formula for nth term = $0.5n^2-1.5n+1$

Working out the n^{th} term for crossover points

Sequence	0 1 3 6 10 15
1^{st} Difference	1 2 3 4 5
2^{nd} Difference	1 1 1 1

Consequently,
A = 0
B = 1
C = 1
The equation for crossovers is = 0+1(n+1) +0.5(n-1) (n-2)1
Simplified = $(0.5n^{2)}$–(0.5n)
Formula for nth term = $0.5n^2$ -0.5n

Working out the n^{th} term for total regions

Sequence	2 4 7 11 16 22
1^{st} Difference	2 3 4 5 6
2^{nd} Difference	1 1 1 1

Consequently,
A = 2.
B = 2
C = 1
The equation for total regions is = 2+2(n+1) +0.5(n-1) (n-2)1
Simplified = $(0.5n^2)$ + (0.5n) +1
Formula for nth term = $0.5n^2$+0.5n+1

Working out the n^{th} term for open regions

Sequence	2 4 6 8 10 12
1^{st} Difference	2 2 2 2 2
2^{nd} Difference	0 0 0 0

Consequently,
A = 2
B = 2
C = 0
The equation for open regions is = 2+2(n+1) +0.5(n-1) (n-2)0
Simplified = 2n
Formula for nth term= 2n

Working out the n^{th} term for the relationship between open and closed regions

Sequence	2 4 5 5 4 2
1^{st} Difference	2 1 0 - 1 -2
2^{nd} Difference	1 1 1 1

Consequently,
A = 2
B = 2
C = 1
The equation for the relationship between open and closed regions = 2+2(n+1) +0.5(n-1) (n-2)1
Simplified = $(0.5n^2) + (0.5n+1)$
Formula for nth term = $0.5n^2+0.5n+1$

Working out the n^{th} term for relationship between open and total regions

Sequence	0 0 -1 -3 -6 -10
1^{st} Difference	0 1 2 3 4
2^{nd} Difference	1 1 1 1

A = 0
B = 0
C = 1
The equation for the relationship between open and total regions = 0+0(n+1) +0.5(n-1) (n-2)1
Simplified = $(0.5n^{2)}-(1.5n+1)$
Formula for nth term = $(0.5n^2)-(1.5n+1)$

Working out the n^{th} term for relationship between total and closed regions

Sequence	-2 -4 -6 -8 -10 -12
1^{st} Difference	2 2 2 2 2
2^{nd} Difference	1 1 1 1

A = -2
B = 2
C = 1.
The equation for relationship between closed and total regions is:-2+2(n+1) +0.5(n-1) (n-2)1
Simplified = -2n
Formula for nth term= -2n

Conclusion

From my results I can gather that my hypothesis were correct. For any particular number of lines the open region was double it. This had to be the case because if a line was to be drawn then it will form a barrier between two regions on either side, i.e. there will be two regions for any one line. Similarly if you were to have two line than the two line will make up four open regions on either side of each of the two line present. Therefore, when two lines were present then the number of open regions was four.

I also predicted that the relationship between open regions and closed regions will be a sequence that has a 1^{st} difference and a 2^{nd} difference and it was. There was also always a greater number of closed regions in comparison to open and finally the number of total regions was also always greater than the number of crossover points.

Evaluation

In my opinion my investigation went quite well. I was able to prove my hypothesis correct and was able to fulfil my aim of working out the nth term for many regions.

It took me a while to draw the diagrams as I wanted each crossover to give me the maximum number of open and closed regions. But once I had the diagrams, I simply put the results in a table and then looked for any obvious or underlying sequence.

Problems and hurdles I faced while undergoing the investigation:

- I had a bit of a problem trying to make sure that I didn't double intersect the lines because if I did that would have ruined all my investigation as it wouldn't have been a fair examination to make sure I didn't do this I double checked all my lines before I went ahead in the investigation.
- I also had a bit of a problem trying to find a complex pattern but after a while I managed to find it.

How would I improve my investigation next time:

- I would give my self a longer time span in order to complete my investigation which will help me enhance the quality of my work.
- I would try to find more complex patterns in order for me to do this I will have analyze all my work more carefully.

Bibliography

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3. CGP intermediate-GCSE maths revision guide / book