

### Initial Formative Assessment Exercise

A function is a relation between a set of input values (the domain) and the output values (the range). Each input must map to a single output. Let  $s(x)$  be the function defined by being equal to the last digit of the value  $x^2$

$$s : x \mapsto (\text{last digit of}) x^2 \quad x \in \mathbb{Z} \text{ values}$$

I will be investigating the output values (the range) of this function and without a calculator, be able to find the output with any integer of  $x$ . Furthermore the function above will be modified i.e.  $s(kx)$  where  $k$  is between 2 and 10. And the ranges identified.

$$s(7) = 9$$

To explain why  $s(7) = 9$  firstly we have to understand the function. The function is defined as if you choose any value (of which is a whole number) for  $x$ , you need to square this number, and after doing so, the value of the function is the last digit of this number.

$$7^2 = 49$$

The last digit of  $49 = 9$

$$\square s(7) = 9$$

Further calculations (without a calculator)

$$s(8) = 4$$

$$s(10) = 0$$

$$s(345) = 5$$

$$s(27,560) = 0$$

$$s(738,954,683,012) = 4$$

Using mental arithmetic I can calculate  $8^2$  and  $10^2$  quite easily. However calculating  $738,954,683,012^2$  is very challenging to say the least. So how can I be confident that my answers are correct? Using the Arabic-Hindu numeral system of hundreds, tens, units etc. and the following simple theory of multiplication I am confident of my answers being correct.

Using a three digit number and each number is represented by letters. Let the first three digits of the first number be 'a' (hundreds digit) and 'b' (tens digit) and 'c' (units digit). And

those of the second number (which for this explanation), we will use the same representation of a, b and c, respectively.

$$\begin{array}{r} a \quad b \quad c \\ * \quad a \quad b \quad c \end{array}$$

The product of these numbers have five parts which are given below and separated by '[' ]'

$$\begin{array}{r} a \quad b \quad c \\ * \quad \underline{a \quad b \quad c} \\ [aa] [aa+ab] [ac+ac+bb] [bc+bc] [cc] \\ \hline \end{array}$$

'aa' is the ten thousands' column, 'aa+ab' is the thousands column etc and the 'cc' is the unit column (math-help-ace, 2008). This is the important part of proving how confident I am of my answers.

□ the square of the last digit of any number given for this function; will determine the last digit.

The range of  $s(x)$  is determined by the squares of the digits 0,1,2,3,4,5,6,7,8,9 as these numbers make up the units column. The squares of the units are **0,1,4,9,16,25,36,49,64,81** respectively. The values in bold are the output values for the function.

□ range of  $s(x)$  □ {0,1,4,5,6,9}

For  $s(x)$  defined above,  $s(2x)$  will be the last digit of the number formed by squaring twice the number  $x$ .

The range of  $s(2x)$  is  $\{0,2,4,8\}$

The range of  $s(10x)$  will be  $\{0\}$

Now to investigate the values of  $k$  for which the range of  $s(kx)$  is the same as the range for  $s(x)$ . To make this easier to show and to determine the results quickly I used Microsoft Excel.

$x$	$s(x)$	$s(2x)$	$s(3x)$	$s(4x)$	$s(5x)$	$s(6x)$	$s(7x)$	$s(8x)$	$s(9x)$	$s(10x)$
0	0	0	0	0	0	0	0	0	0	0
1	1	2	3	4	5	6	7	8	9	0
2	4	8	2	6	0	4	8	2	6	0
3	9	8	7	6	5	4	3	2	1	0
4	6	2	8	4	0	6	2	8	4	0
5	5	0	5	0	5	0	5	0	5	0
6	6	2	8	4	0	6	2	8	4	0
7	9	8	7	6	5	4	3	2	1	0
8	4	8	2	6	0	4	8	2	6	0
9	1	2	3	4	5	6	7	8	9	0
Range	0,1,4,5,9,6	0,2,4,8	0,2,3,5,7,8	0,4,6	0,5	0,4,6	0,2,3,7,8	0,2,8	0,1,4,5,6,9	0

Figure 1. Showing the range of the function  $s(kx)$  (where  $k = 2,3,4,\dots,10$ )

Figure 1 shows that the range of  $s(9x)$  is the same as the range for  $s(x)$ . So the answer to the question, investigate the values of  $k$  for which the range of  $s(kx)$  is the same as the range for  $s(x)$  is 9.

Introducing a new variable  $y$ , I am now going to investigate when  $s(x + y) = s(x) + s(y)$  where  $y$  is an integer and  $s(y)$  is defined in the same way as  $s(x)$  above. When thinking of how to display and calculate the answer the easiest method was to calculate each part of the equation and then compare results and match the answers for which they have the same output. These are displayed on the following pages 5 and 6. Figure 2 shows the output values for  $s(x + y)$  and Figure 3 shows  $s(x) + s(y)$ .

To make things a little clearer with these two tables I have highlighted the values of which the output values are the same. These tables show that for when  $x = 0$ ,  $y$  can equal any value between -9 and 9. It also displays that for when  $x = \pm 5$ , then  $y$  can equal any value between -9 and 9. This is also true for the reverse; when  $y = 0$ ,  $x$  can be any number between -9 and 9, and when  $y = \pm 5$ ,  $x$  can equal any value between -9 and 9. When I am mentioning  $x$  or  $y$  being a number between -9 and 9, I am actually talking about the last digits of the integers used.

$$\square \ x = 0, y \square \square, -9 \leq y \leq 9$$

$$\square \ x = \pm 5, y \square \square, -9 \leq y \leq 9$$

The values  $x$  and  $y$  are commutative for the values mentioned above. The range of  $x$  and  $y$  when  $s(x + y) = s(x) + s(y)$  is  $[0, 1, 4, 5, 6, 9]$ .