For a duopoly involving homogeneous products, explain and contrast a Cournot, Stackelberg and Bertrand equilibrium.

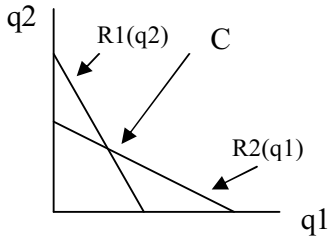
The critical problem faced by a firm in an oligopoly is that its decisions affect the prices and quantities of its rivals. The oligopoly problem arises because, where there are only a few suppliers to the market; the demand for the product of one firm depends significantly on the price and output. A non–cooperative duopoly is an industry consisting of two firms in which firms take their decisions independently and can be classified according to whether firms treat quantity or price as the key strategic variable. When it comes to quantity setting there are two major models put forward. The first, developed by Cournot in 1838 is based on firms setting quantities simultaneously where each firm is setting the output that maximises its profit given the output of its rival. In 1934 Stackelberg argued that one firm takes the role of 'leader' with the other firm acting as a 'follower' emphasising the quantity leadership view. Here the leader anticipates the response of the follower and uses this to its own advantage.

Bertrand in 1883 argued that price, not output, should be the firms decision variable where rivalry between the duopolists would result in both setting a zero price. Each of the models provides a different equilibrium output and welfare level

We assume a linear market demand curve, P(Q) which is given by a – Q where a is a positive parameter. Further we assume that all firms would incur the same constant per unit production cost, c, where c < a and for simplicity we follow Cournot in supposing that there are no costs of production (c = 0).

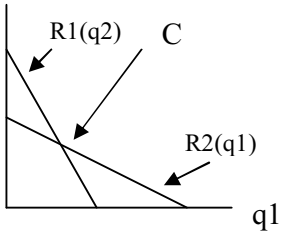
The Cournot model puts forward a case for simultaneous quantity setting where at the beginning of each period the firms take their decision independently and simultaneously. Here the profits of firm f (same as TR) will depend on both outputs which is given by $\prod 1 = q1(a - q1 - q2)$. The interdependence between the 2 firms results in the profit-maximising output for firm 1 depending on the output of firm 2 and vice versa which is modelled using reaction function/curves. Firms 1's reaction curve q1 = R1(q2) shows that the profit maximising output of firm 1 is a function of the output of firm 2. As production costs are zero and the assumed market demand, the specific form of the reaction function is linear over the relevant range R1(q2) = (a-q2)/2. To confirm 1's optimal response to q2 we partially differentiate $\prod 1$ (q1,q2) with respect to q1 to obtain the first order condition:

Further reaction curves can be derived using iso-profit curves which show the output combinations that yield firm 1 the same profit. The can be used to show firms 1's optimal response to q2. Where the slope of the iso-profit curves are zero we get a line going through to indicate the firms reaction curve. Asthe model involves symmetry between the firms, the reaction curve for firm 2 over the relevant range is given by q2 = r2(q1) = (a – q1)/2



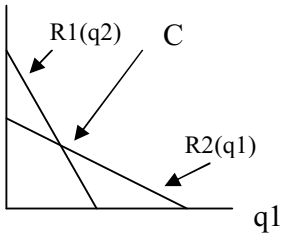
This diagram shows the two reaction curves for firm 1 and firm 2. An output combination (q1c, q2c) is a Cournot equilibrium if and only if each firm is maximising its profit given the output choice of the rival firm, $q^c_1 = R_1(q_2^c)$ and $q^c_2 = R_2(q^c_1)$ that is each firm is on its reaction curve.

In a Cournot equilibrium firms choose their outputs simultaneously so a firm chooses it output not knowing the output of its rival firm, and thus a firm does not 'react' to its rivals output. A Cournot equilibrium can be interpretated as a state of rest for a dynamic process in a multi-period model in which each duopolist bases its own decision on the expectation that its rival produce the same output as in the previous period. The intersection of the reaction curves is such a state of rest where each firm is maximising profit given the output of the other firm and since each firm believes its rival will continue to produce that output, it has no incentive to vary its own output. This means in a Cournot Equlibrium, each firm is maximising its profits given its expectations of the output choice of the other firm, which are confirmed.



This process will always converge on the Cournot equilibrium as can be seen on the diagram. If firm 1 is the only firm in the industry it will produce monopoly output. As firm 2 enters the next day it will base its output decision on firm 1's monopoly output. Firm 1 will continue to produce its monopoly output not anticipating firm 2's entry. Thus day 2 output combination is on firm 2's reaction curve but not on firm 1's reaction curve. On day 3 firm 2 will expect firm 1 to continue its previous output level so keeps it own output level unchanged. Yet firm 1 cuts its output, this being a lagged response to the entry of firm 2. Therefore day 3 output combination is on 1'd reaction curve but not on 2's reaction curve. Given the shapes of the reaction curve as the process continues it converges on the Cournot equilibrium which is a stable.

Further support of this equilibrium position comes from the single period game-theoretic model in which it is common knowledge that both firms have complete information and that both firms are rational. A distinct feature of a Cournot equilibrium-termed the Cournot-Nash equilibrium, is where each firm maximises its profit given its expectation of the output choice of the other firm. To argue that the Cournot equilibrium holds both firms would expect its rival to produce at the equilibrium and thus the firms are rational.

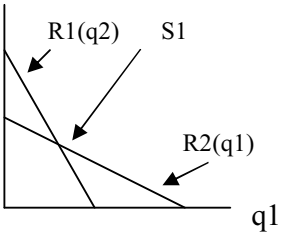


This process uses elimination of expectations to arise at the equilibrium. It would not be consistent with it being common knowledge that both firms are rational for firm 1 to expect firm 2 to produce q2. Firm 1's reasoning would be that firm 2 will not produce q2 because firm 2 would only do so if firm 2 expected firm 1 to produce q1. But firm 2 knows that firm 1 would not produce as a high quantity as that even if firm 1 were a monopolist. Further, firm 2 will not produce q2 because firm 2 would only do so if firm 2 expected firm 1 to produce q1. Firm 2 knows that firm 1 would not produce q1 if firm 1 expected firm 2 to produce more than the monopoly output and firm 2 knows that firm 1 would not expect it to do that.

Therefore the Cournot output combination constitutes the solution in that it is the only output combination based on expectation that cannot be eliminated by the foregoing type of argument, that is, on expectations that are consistent with it being common knowledge that both firms are rational.

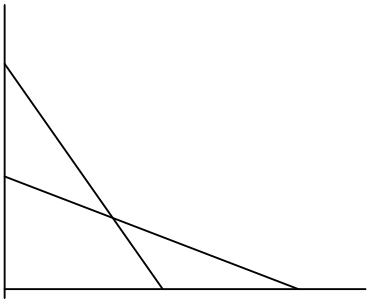
The quantity leadership theory developed by Stackelberg produces two versions which provides a different equilibrium output to Cournot. The first, a game-theoretic single period model assumes duopolists take their decisions independently but in a well defined sequence. Firm 1 the 'leader' chooses its output and firm 2 the 'follower' having observed firm 1's choice decides on its own output. The leader has first mover advantage because firm 1 knows that firm 2 will observe firm 1's output choice before deciding its own output. Therefore firm 1 anticipates firm 2 reaction to its own choice. There is a first mover advantage which results in higher profit for the leader (firm1), over the follower (frim2).

Firm 1 needs to choose the output combination on firm 2's reaction curve R2(q1) that maximises firm 1's profit therefore, the output combination is the point of tangency between the iso-profit curve denoted by $\prod_1^{s1}$ and R2(q1). If the two firms swapped roles the industry output and price would be the same.



This diagram shows the Stackelberg equilibrium

If we compare the Stackelberg equilibrium (S1) where firm 1 is leader and firm 2 is follower, and vice versa and compare this with the Cournot equilibrium, we see that firm 1's profit is greatest when firm 1 is a Stackelberg Leader and lowest when firm 1 is a Stackelberg follower. The diagram below shows this



The leader has a first mover advantage in two senses: firm 1's profit as Leader is not only greater than it would be if firm 2 were Leader but also greater than it would be for a Cournot equilibrium where both firms take their decisions simultaneously. The follower is disadvantaged by knowing firm 1's choice before making its own choice-moreover it is disadvantaged by the fact that firm 1 knows that firm 2 will have this information.

The more traditional second version of the Stackelberg treats leadership and following as alternative behavioural modes. The follower behaves as a naïve Cournot duopolist, the leader realises that the follower reacts naively to its own output decisions

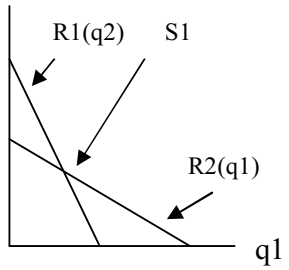
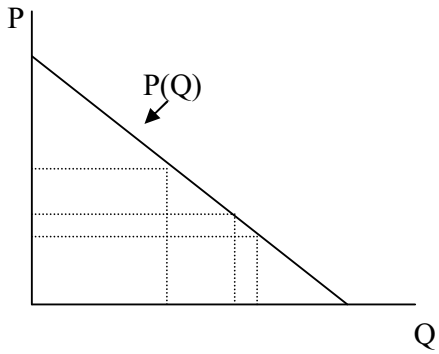


Figure 5a shows that S1 here depicts a state of rest for this version of the Stackelberg model. Firm 2 (the follower) is maximising its profit given its (Cournot) belief that firm 1 will continue to produce q_1^{s1} and firm 1 the leader is maximising its profit given its belief that firm 2 will react according to R2(q1). Therefore neither firm would vary its output.

With this behavioural modes characteristic , the Cournot model is a special case of a more general Stackelberg model, the special case where both firms act as followers. If both firms acted as Stackelberg leaders the result would be an unpredictable price war.

If we compare the Stackelberg and Cournot equilibriums on one diagram we see that the Stackelberg equilibrium provides a higher level of output compared to the Cournot equilibrium yet it is lower than the perfect competition level of output. Both Cournot and Stackelberg outputs are higher than that produced by the monopoly. In terms of price the Cournot model produces a higher price level compared to the Stackelberg model. Both price levels are higher than the perfect competition price yet lower than the monopoly price.



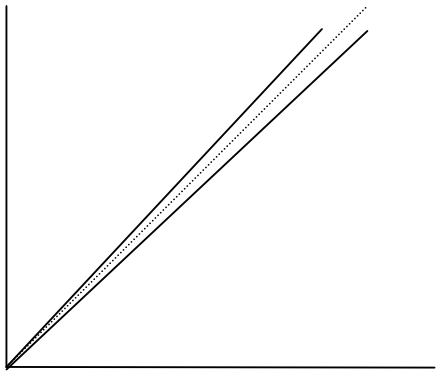
The Cournot level of profit is also greater than the Stackelberg profit level yet when it comes to social welfare the Stackelberg model outweighs the Cournot model. Shifts from the monopoly equilibrium to Cournot equilibrium results in lower industry profits and would be outweighed by gains to consumers.

In a single period context, even though firms make a verbal agreement to split equally the monopoly output, the agreement would not be self-enforcing, as each firm has an incentive to violate the agreement itself. i.e. Qm/2 is not the optimal response to the other firm producing Qm/2. The only self-enforcing agreement would be one stipulating the Cournot output (q1c, q2c). Here neither firm would have an incentive to break the agreement and neither would have reason to expect its rival to break the agreement.

The quantity level of output differ very much to that of price setting led equilibrium quantities by Bertrand. Bertrand argued that duopolists independently and simultaneously set prices, p1 and p2. As there is homogeneous products, consumers will always choose the firm setting the lowest price. Thus with the unique Bertrand equilibrium/Bertrand-Nash equilibrium.

$p1 = p2 = 0$ $q1 = q2 = a/2$ $q = a.$

Even though there are only two firms, equilibrium involves the perfectly competitive industry output and price, which differs greatly from the two previous models we have considered and is termed the '***Bertrand Paradox'.*** The Bertrand equilibrium may be interpreted as a state of rest for a dynamic process in a multi-period model in which each firm expects the other firm to charge the same price as in the previous period. P1 = P2 = 0 will be the state of rest because neither firm would raise its price (since it would expect to lose all its customers). If firm 1 was charging a higher price than firm 2, firm 1 would expect to take all the customers if it lowered its price to just below the price of firm2 (p1>p2>0). If both firms were charging the same price but this was greater than zero, each firm would expect to benefit by reducing its price slightly below its rival. Further if firm 1 was pricing at a price greater than firm 2 which was greater than zero, firm 2 would expect to benefit by raising it price. There is no incentive for firm 1 to lower its price but there is an incentive for firm2 to increase its price close to p1. Therefore the only state of rest can be when p1 = p2 = 0. The Bertrand equilibrium is the soln in a single period game-theoretic context, as the only expectation consistent with it being common knowledge that both firms are rational are for firm 1 to expect firm2 to set $P^B_2=0$ and for firm1 to set $P^B_1=0$.



Bertrand price competition is sometimes presented in terms of price reaction curves. Firm 1's price reaction curve shows firm 1's best price in response to firm 2's price. **The intersection constitutes a Bertrand equilibrium.** The dotted line represents p1 = p2. Pm is the monopoly price and a firm price cannot exceed this price. The reaction curves should be interpreted as being close to the 45 degree line, reflecting the idea that a firm would set a price just below its rival price. A firms best response is not well defined in the case of identical products.

When it comes to price leadership firm 1 sets its price p1 and firm 2 observes p1 and sets its price p2. if firm 1 sets a positive price p1, the best response for firm 2 would be to set a price p2 = p1 – e, where e is 'small' and thereby take all the customers. To this extent there might seem to be a 'second mover advantage.' However firm 1 anticipating firm 2's response would be obliged to set a zero price. Thus the equilibrium would involve zero prices. As the firm knows that it will be undercut when it sets a +ve price and therefore sets price=0. With differentiated products, the equilibrium of price leadership is consistent with +ve profits. Since there is a 2nd mover advantage, why would one firm elect to set price before the other firm does so.