"Two key aspects of the investment decisions of firms are that they are undertaken in a cloud of uncertainty, and that investment spending over the business cycle is highly volatile. Concentrating on the investment decision as the balance between the return and the cost of the investment does little towards a useful economic explanation". Examine q-theory and other theories of investment. How well do they explain investment decisions?

Most of the literature that focuses on the behaviour of investment either tends to focus on the relationship between the return and cost of capital or the relationship between investment and output. With regard to the former, the most basic neoclassical theory looks at investment decisions under a situation of perfect competition. There is no uncertainty, a perfect elastic supply of capital goods and the firm can adjust their capital stocks costlessly. If these assumptions hold the firms profits can be written as follows:

$$\pi(K, X_{1,\ldots}\ X_n) - r_K K$$

In the above equation, 'K' is the amount of capital a firm rents, the X variables are exogenously given so as to include the cost of other inputs and the price of the firms product, and 'r_K' is the rental price of capital. By taking the first derivative of the above equation, the firm rents capital until its marginal revenue product equals its rental price - a balance between the return and cost of capital.

However, one can extend upon this analysis by replaced the rental price of capital with the 'user cost of capital'. As most capital is not rented but is owned by firms, it is necessary to take into account the opportunity cost of the investment decision (the interest received if it didn't invest), the level of depreciation (given by the depreciation rate 'δ'), the changing price of capital and the tax system. However, even if one extends the analysis to take all these factors, the investment decisions is still a balance between the cost and return of capital under a competitive market framework. Although this seems to have the natural economic intuition of profit maximising behaviour, the simple neoclassical theory is of little use in explaining the observable investment behaviour in the economy. The volatile nature of investment decisions cannot be described by changes in the user cost of capital, and moreover, most research and surveys conducted points toward the role of future expectations in determining current investment decisions. From a theoretical point of view, the underlying assumptions of the model, especially the ability of firms to change their capital stock costlessly, are argued to be too restrictive on the model.

The 'q-theory' of investment tries to resolve some of these issues by taking into account the adjustment costs involved with changing the level of capital. It also provides a neat analytical framework under which the role of uncertainty can be added (although this will be dealt with later). The adjustment costs of capital can be divided into tow categories: internal and external. Internal adjustment costs are the direct costs of changing the capital stock, such as the installation costs involved with new capital. External costs are related to the supply of capital to a firm - the supply of capital is not perfectly elastic so that changes to the firms' desired capital stocks raises the price of capital.

Before the model can be outlined, one must first look at the underlying assumptions. The market is deemed to be composed of 'N' identical firms. The production function must have constant returns to scale (linear homogeneity), output markets are competitive (the firm is a price taker), and the supply of all factors of production apart from capital is perfectly elastic. Given these assumptions, the firm's profits are proportional to its capital stock

(κ). If one adds the assumption that the demand curve for industry's product is downward sloping, profits can be said to be a decreasing function of the industry-wide capital stock (K). Central to this model is the concept of adjustment costs. For this analysis, one will only consider internal adjustment costs, which rely on the assumption that the costs are a convex function of the rate of change of the firm's capital stock. This implies that it is costly to change the capital stock and the costs are greater, the greater the size of the adjustment. It is worth noting that later the assumption of constant returns (or linear homogeneity) in the adjustment cost function will be introduced. Finally, for simplicity, the purchase price of capital goods will be set to 1 (i.e. only internal adjustment costs), the depreciation is equal to zero, so that the change in the capital stock is equal to the investment (I) in time 't', and that the real interest rate (r) is constant.

The assumptions outlined above imply the that firm's profits at a point in time are given by:

$$\pi(K)\kappa - I - C(I)$$

Therefore, the firm maximises the present value of these profits when:

$$\prod = \int_{t=0}^{\infty} e^{-rt} \left[\pi(K(t))\,\kappa(t) - I(t) - C(I(t)) \right] dt$$

By the application of some calculus, one can derive various conditions of optimising behaviour. If one defines the variable 'q' as the value to the firm of an additional unit of capital, then:

$$1 + C'(I(t)) = q(t)$$

This condition shows that the cost of acquiring a unit of capital equals the purchase price (fixed at 1) plus the marginal adjustment cost; thus the firm invests to the point where the cost of acquiring capital equals the value of capital. In accordance with previous theories, investment is still a balance between the return and cost of capital.

The second condition that can be derived is that:

$$\pi(K(t)) = rq(t) - \dot{q}(t)$$

The left-hand side of the above equation is the marginal revenue product of capital, and the right hand side is the opportunity cost of capital. For the firm to be maximising the returns to capital must equal its opportunity cost. The equation shows that the opportunity cost of capital involves two parts; owning a unit of capital for a period means that a firm forgoes 'rq_t' of real interest and offsetting capital gains of the change in the value of capital stock. The final condition is known as the 'transversality condition', which is that the value of the firm's capital discounted must approach zero. If this condition holds then the value of a unit of capital at a given time is given by:

$$q(t) = \int_{\tau=t}^{\infty} e^{-r(\tau - t)} \pi(K(\tau))\, d\tau$$

This equation states that the value of a unit of capital at a given time equals the discounted value of its future marginal revenue products. The implication of these conditions of q-theory is that investment will take place

wherever the net discounted value of an investment is positive (a balance of return and cost over many time periods).

The model can be diagrammatically analysed using a phase diagram in (q, K) space. As the market purchase price of capital is set to 1, it is possible to draw a function where there are no changes to the capital stock (where C'(I) = 0). Moreover, it is also possible to draw a function where 'q', the market value of an additional unit of capital to its replacement cost, does not change. Since 'π(K)' is decreasing in K, due to the downward sloping demand curve for the industry's product, the loci of points where 'q' does not change is downward sloping in (K, q) space. This can all be shown in the diagram below:

Using the above phase diagram, it is now possible to analyse the dynamics of investment. In other words, the model suggests a way to explain the volatile nature of investment behaviour. The above diagram shows that investment will always tend to the equilibrium level of capital stock 'E' as long the initial levels of q and K obey the transversality condition. If the market of value of capital is high relative to its replacement cost, investment will take place, and one will move along the saddle path to the new equilibrium at E. The reverse can also be shown if the capital stock K is too high. The higher one is above the $\dot{K}=0$ function, the higher the level of investment; the faster new capital is being added to the existing capital stock.

The phase diagram provides a useful tool of analysis to examine the dynamics of investment behaviour. Changes in aggregate output, interest-rate movements and changes to the tax system can all be analysed under a 'q-theory' framework. It is now worth discussing in more detail the economic intuition behind the value of 'q'. So far, it has been shown that the market value of a unit of capital is given by 'q'. Moreover, a unit increase in the firm's capital stock increases the present value of the firm's profits by 'q', and thus raises the value of the firm by 'q'. As the purchase price of capital has been equated to one, 'q' is therefore the ratio of the market value of a unit of capital to its replacement cost. This ratio is known as 'Tobin's q' (1969). However, it is important to note that so far that the ratio is revelant to marginal q. As Cuthbertson and Gasparro argue 'marginal-Q is a sufficient statistic for investment but is unobservable'. Therefore, if one assumes a linear homogeneity (constant returns) in the adjustment cost function, average q equal to marginal q, and average q is directly observable. Average q is the ratio of the total value of the firm to the replacement cost of its total capital stock.

In assessing the model, much research has been devoted to the relationship of average 'q' and investment. However, despite the ability of the model to theoretically explain dynamic movements in investment behaviour, empirical evidence shows that 'average-Q does not provide a satisfactory explanation of UK or US fixed investment using either aggregate time-series or panel data' (Cuthbertson and Gasparro). If average 'q' is not a sufficient explanation for fixed investment, and marginal 'q' is not directly observable, perhaps it is worth analysing other theories of investment. As highlighted before, a large amount of literature has been devoted to the relationship between investment and output. Many of these 'accelerator' theories look to find an empirical relationship between the growth of output and the investment, often in a lagged form. Whilst empirically, the relationship ' provides a reasonable statistical explanation of investment', the mechanisms of causation are not clear. Is there a relationship of causation or are they just correlated variables? Precious (1987) provides an interesting insight into the role of demand constraints on firm's investment decisions. By making a break from the neo-classical assumptions of perfectly competitive markets, it is possible to argue that 'only when the firm is demand constrained or perceives it will be demand constrained in the future, is investment determined by output'. Cuthbertson and Gasparro (1995) construct a model in which investment is determined by average-Q, capital gearing (the ratio of debt at market value to the value of the capital stock at replacement cost), and for those firms that are demand constrained, by output and real wages. In other words, a large combination of factors from q-theory and accelerator models is taken into account. They find that for the period 1968-1990, their model provides an 'adequate model for aggregate UK manufacturing investment'.

So far the analysis of investment theories has avoided the role of uncertainty in decision making. Survey evidence, such as that from the CBI, all point to the role of uncertainty as a determinant in investment decisions. In his 'General Theory', Keynes examines the role of 'long term expectation'. Many of the ideas behind the theories that take uncertainty into account appear in this chapter. Whilst Keynes looks at investment decisions being determined by the marginal efficiency of capital (MEC) schedule, he argues that this MEC is affected by the state of expectations. He examines the means by which speculation causes the stock prices not to represent 'fundamentals' of value of capital assets, as individuals try to 'beat the gun'. Perhaps more importantly, with regard to fixed investment, he examines the role of 'animal spirits…the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than on a mathematical expectation'. This is not to suggest that investment behaviour is dependent on irrational psychology, but that investment is not strictly depend on return and cost.

Another factor worth considering under uncertainty is the role of financial constraints. The traditional neo-classical models of investment assume that firms operate in perfect capital markets, so that they can borrow or lend as much as required at a given rate of return. If this holds, external finance (e.g. the issuing of new shares) should be a perfect substitute for internal finance from retained profits. However, as Bond and Jenkinson point out, survey evidence shows that as many firms report that their investment is constrained by the availability of internal finance, the assumption of this assumption must be questioned. A significant amount of research has been devoted to the idea that suppliers of external finance (due to uncertainty) have less information about the quality of the firm's investment opportunities, so external finance is perceived to be more expensive by firms.

Finally, it is possible to extend upon q-theory to take into account the role of uncertainty. It can be shown that if the profit function is linear and the adjustment cost function is quadratic, then uncertainty does not affect investment: firms invest as long as the value of new capital (given by expected payoffs) exceeds the cost of capital. However, once these assumptions are removed, uncertainty about profits can affect current investment. The notion of 'irreversible investment', where it is more costly for firms to reduce their capital stocks (through depreciation)

than to increase them, has become extremely important in describing investment under uncertainty. If adjustment costs are asymmetric, the saddle paths in the diagram below become concave. Therefore, given a change in the exogenous variables, such as the possibility of a new tax policy, or an interest rate reduction will cause the increase in investment to be lower than under symmetric costs. To add the economic intuition, if investment is irreversible, not investing today leaves the firm with an option to expand later, should expansion prove to be necessary. Investing in the present time period eliminates this option, and the loss of this option can be considered as part of the cost of investing.

In the above diagram, before the expected change leads to a movement to B, which due to it's concave nature, means a lower level of investment than what would have been if investment was reversible. Once the outcome of the change in tax or real interest rates is realised, investment moves onto the appropriate saddle path, associated with the new equilibrium

To conclude, many theories of investment focus on the balance between the return and cost of investment. In particular, q-theory has become very popular in order to explain the dynamics of investment behaviour. However, if one imposes the assumption of constant returns in the adjustment cost function, the observable average q is equal to the marginal q. However, empirical research does not support this restriction. Instead, factors such as output provide a more useful explanation. Moreover, once uncertainty is taken into account, more useful economic explanations can be found; internal finance and irreversible investment are both useful in explaining investment behaviour. Therefore, it is possible to argue that return and cost do play a vital role in determining investment decisions, but other properties must be taken into account.

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