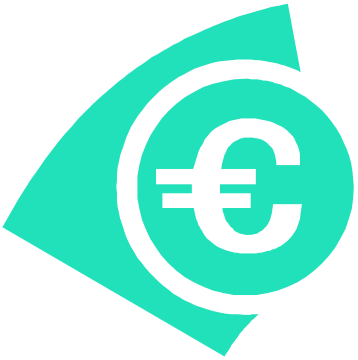

Portfolio Diversification

Shalenta Hardison
Fatih Akduman
Andro Bibileishvili

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1. Executive Summary

Portfolio Diversification deals with choosing a group of assets with very low correlation to one another in order to reduce the risk of investing. An investor's incentive for putting their capital into certain markets for a particular length of time is done in order to receive a return on their investment. Managing the level of return and the degree of risk is also another challenge confronted by an investor. In order to optimize the return on investment, an investor has to first determine whether or not they are risky investor or risk-adverse. After this has been determined, an investor can gauge the amount of return they can potentially receive in. An investor must also determine whether they are looking for a short-term investment or a long-term investment. This will also determine how much capital the investor wants to place in any one asset.

Since money markets are considered inefficient and unregulated by the actual investor, an investor has to apply diversification techniques to mitigate the market risks associate with investing. Throughout the report, it is important to note that both risk and the rate of return share a positive correlation. As an investor, the higher the risk associated with the asset, the higher the potential for a greater rate of return. This report will look at key terms associated with portfolio diversification, relevant theories and applications, the limitation of relevant theories, pertinent risk and return calculations, optimizing an investor's portfolio and general conclusions.

2. Key Terms

Portfolio - a combination of investments, securities or physical assets and placing them into a single 'bundled' investment.

Portfolio Diversification- choosing a variety of asset classes with low correlation to each other to reduce exposure to risk. (E.g. stocks, bonds, cash, T-bills, real estate, mutual funds etc.)

Correlation- the degree to which an asset moves in relation to another on the market. Co-related assets= correlation of +1 (BAD) Negatively co-related assets= correlation of -1 (GOOD)

Asset Allocation- deciding which broad category of assets to include in a portfolio based on time horizon and risk tolerance

Risk Tolerance - a measure of willingness to accept higher degree of risk in exchange for the chance to earn a higher rate of return

Investment Horizon- number of years you have to save for a financial goal

3. Theories and Applications

3.1 Perfect Portfolio Effect- Investing in Apples and Pears

In the Perfect Portfolio Effect, an investor can choose to invest in apples, pear or both. In this particular situation, both apples have the desired negative correlation of -1, when one moves up on the market; the other acts reciprocal of the other. The examples below show the different return rates for both apples and pears. Although each fruit will potentially yield a return of 10%, it is in the best interest of the investor to diversify the risk in a variation of ways. The follow illustrations show the different combination of investments that can be made in relation to apples and pears.

Apples:

50% of yielding 8% return and 50% of yielding 12% return

Pears:

50% of yielding 6% return and 50% of yielding 14% return

Apple investment: ER= (0.5 *8%) + (0.5*12%) = 10 %

Or

Pear investment: ER= (0.5* 6%) + (0.5* 14%) = 10%

In the above example, the risk of both Apples and Pears has not been decreased. Instead, the investor takes on the risks that are associated with market activity, i.e. inflation, decrease in demand etcetera and makes the decision to invest in only one asset. In either situation, the investor may yield more or less than the 8% for the first half of year for Apple and 12% for the second half of the year. The same theory applies to Pears. Keeping this in mind, the investor might want to invest in the following if they are a risk-averse investor.

Apple investment (0.5 *8%) + Pear investment (0.5* 6%) = 7%

If an investor classifies themselves as a risky investor, they would invest in the following based on the past historical activity of the particular asset.

Apple investment (0.5*12%) + Pear investment (0.5* 14%) = 13%

The last two situations illustrate alternative investment options the investor could entertain. In either situation, there are still risks associated with the investment. On the capital market, investors can on estimate the future returns of stocks based on historical performance measures and current market activity.

3.2 Modern Portfolio Theory

The original theory was developed by Markowitz in 1952 and focused on:

- Calculating risks
- Reducing risks
- Optimizing rate of returns for investors

According to the theory portfolio performance is determined by the following five determinants:

- Time horizon
- Risk tolerance
- Investment goals
- Financial means
- Level of investment experience[1]

Modern Portfolio Theory (MPT) proposes how rational investors will use diversification to optimize their portfolios. According to MPT it is possible to construct an "efficient frontier" of optimal portfolios offering the maximum possible expected return for a given level of risk[2]. There are four steps for portfolio construction:

- Security valuation
- Asset allocation
- Portfolio optimisation
- Performance measurement

MPT is based on the following assumptions. First, it is in respect to mean and variance**.** It has been assumed that investor's risk/reward preference can be described

[1] www.investopedia.com viewed on 28.03.2006
[2] Corporate Finance and Investment, R. Pike and B. Neale, Fourth edition, p.318

via a utility function. Only the expected return, in other words the mean return, and the volatility, meaning the standard deviation matter to investor.

Secondly, there is the assumption about risk and reward, which states that investors are risk averse. Hence investor will take on increased risk only if compensated with higher expected returns.[3]

MPT states that apart from looking at expected risk and return of one particular stock, investing in more stocks can give the benefit of diversification ("Not putting all your eggs in one basket"). Each stock has its standard deviation from the mean, called risk. Markowitz showed that investment is not just about picking stocks but about choosing right combination of stock. Markowitz identifies two kinds of risk:

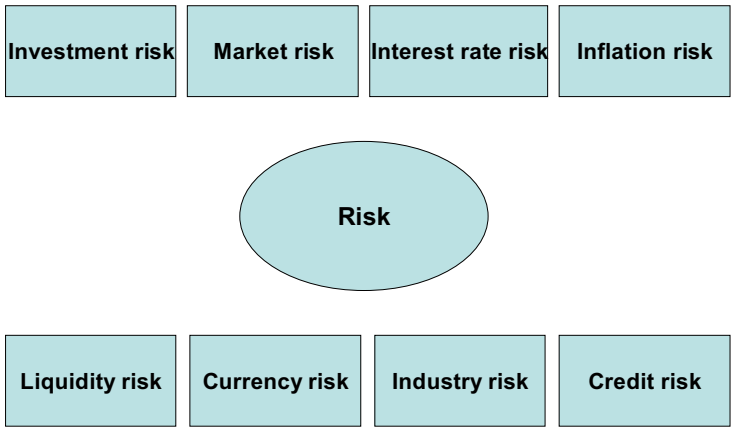
- Market risk (=Systematic risk)
- Specific risk (=Unsystematic risk)

3.2.1 Risks

Risk is defined as "the uncertainty that you may not earn your expected return on your investment."[4]There is a distinction between market risks and specific risks. Market risks are risks that can not be diversified away, for example interest rates or recessions. Specific risk is a risk that is specific to individual stocks and can be diversified away as number of stocks in portfolio increases.[5] There are eight types of risks as shown on the diagram below.

[3] www.wikipedia.org/wiki/Modern_portfolio_theory viewed on 26.03.2006
[4] www.partners.financenter.com viewed on 27.03.2006
[5] Corporate Finance and Investment, R. Pike and B. Neale, Fourth edition, p.336

Types of Risk



Investment risk - Risk that the investment value will fall

Market risk - Risk that the entire market where investments are traded will fall in value

Interest rate risk - Risk that interest rates change while holding an investment and higher rates result in lower returns on stocks and bonds

Inflation risk - Bonds are especially vulnerable to this risk, whereby stocks are less sensitive since dividends can be adjusted to inflation

Liquidity risk - Chance that stock or bond investment can not be sold easily due to lack of buyers (→ thinly traded security)

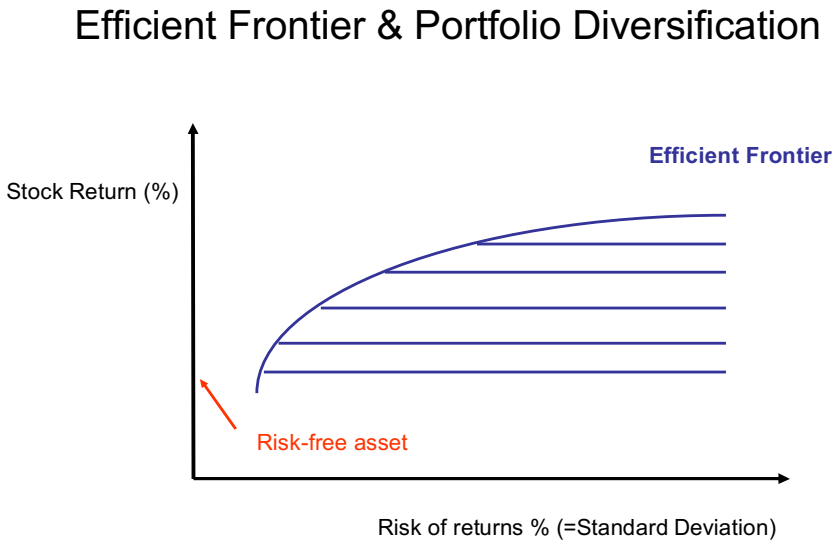
Currency risk - When buying a company' stocks or bonds, one buys piece of its operations. If it sells products abroad same currency risk is faced-unless it is hedged

Industry risk - Chance that a set of factors drags down overall industry performance, For example cold weather affecting retail industry

Credit risk - Chance that the company selling bonds defaults on debt payments[6]

3.2.2 Efficient Frontier & Portfolio Diversification

The best level of diversification is determined by the Efficient Frontier. For every level of return, there is one portfolio that offers the lowest possible risk, and for every level of risk, there is a portfolio that offers the highest return. These combinations can be plotted on a graph giving us the **"Efficient Frontier"**



[7]

Any portfolio on the upper part of the curve is efficient, in other words it gives maximum expected return for a given level of risk. Rational Investors will only hold portfolios that lay somewhere on the efficient frontier. Maximum level of risk that an investor will take on determines the position of the portfolio on the line. Risk-free assets are investments in e.g. state bonds. MPT suggests that combining a stock portfolio with a risk-free asset can increase returns beyond the efficient frontier. However, MPT has also limitations, which are:

[6] http://partners.financenter.com viewed on 28.03.2006
[7] www.investopedia.com viewed on 27.03.2006

- It often requires investors to rethink notion of risks, since sometimes it demands investors to take on a perceived risky investment (e.g. futures) to reduce overall risk
- It assumes that it is possible to select stocks whose individual performance is independent of other investments in the portfolio
- It is logical to borrow to hold risk-free assets and increase portfolio returns, but although government-backed bonds are free of default risk, interest rate and inflation rate risks remain

3.3 The Sharpe Ratio

"A risk-adjusted measure developed by William F. Sharpe, calculated using standard deviation and excess return to determine reward per unit of risk. The higher the Sharpe ratio, the better the funds historical risk-adjusted performance."[8] Efficient Frontier determines the most risk-efficient portfolios for a given collection of securities; Sharpe ratio goes beyond by helping to identify the best possible proportion of these securities to use in combination with a risk-free asset.

In mathematical terms, the quantity known as the Sharpe ratio represents a measure of the amount of additional return a portfolio provides compared to the linked risk .

It is defined as:

$$\mathbf{S(x) = (R - RF) / \sigma(x)}$$

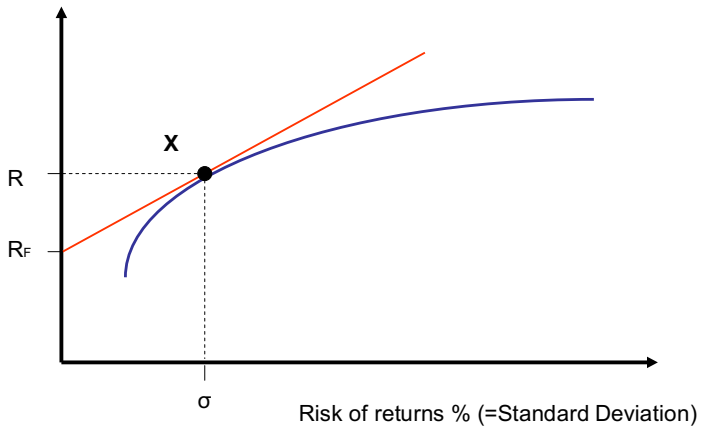
Where x is some investment

R is the average annual rate of return

RF is the best available rate of return of a risk-free security

σ is the standard deviation of R

The Sharpe Ratio



[8] http://www.investorwords.com/4536/Sharpe_ratio.html viewed on 27.03.2006

Looking at the diagram above, it should be noted that taking some investment like "x" and combine it with cash, the resulting portfolio will lie somewhere along the straight line joining cash with "x". Secondly, to maximize the rate of return, select "x" that gives the line with the greatest possible slope. And finally, the slope of this line is equal to the Sharpe Ratio of "x".[9]

3.4 Capital Asset Pricing Model

The Capital Asset Pricing Model was developed in 1964 by Sharpe; the theory is used to explain how efficient capital markets value securities by discounting future expected returns at risk-adjusted discounted rates.

Model explains how the capital market sets share prices and the importance of measuring risk and setting risk premiums to each asset. This model also explains the relationship between risk and the rate of return and is used in the pricing of securities[10]. The following assumptions are made for this theory:

- All investors operate on a common single period planning horizon.
- All investors are rational and risk-averse.
- There are no transaction costs in trading securities.
- All securities are highly divisible.
- All investors select from investments by looking at expected return and risk.
- No one investor can influence the market price by their own transactions.

3.4.1 Security Valuation and Discount Rates

In order for a shareholder to receive a certain rate of return for their initial capital investment, a company must first assert a value to the stock being issued. Pike and Neale give a clear example of stock valuation which is illustrated below

Example:

Company X issues £1 mil shares for a 1 yr project offering a cash flow of £10 million at 25% discount rate:

V0	= £10m/(1.25)	= £8m
Market price	= £8m/£1m	= £8/share

[9] www.moneychimp.com viewed on 27.03.2006
[10] Corporate Finance and Investment, R. Pike and B. Neale, Fourth edition, p.326

This example takes the company's projected cash flow from the investment made (either in Debt or Equity) and issues a certain pound amount in shares made available on the capital market. A discount rate is added to the valuation of the stock in order to determine the percentage required return needed to convert the expected return into the present value. In other words, a discount rate is synonymous with an interest rate used to determine the present value of future cash flow.

3.4.2 Discount Rate- Three Major Components

The discount rate is the percentage required return used to convert future expected cash flows into their equivalent present values. The discount rate for an asset looks at three components, the allowance for risk, the allowance for time value of money and the allowance for price changes in the market. Theses allowances have been described in more detail below:

- **Allowance for risk**- the promised reward that provide and incentive for investors to expose capital to risk.
- **Allowance for time value of money**- compensation to investors for having to wait for their payments.
- **Allowance for price changes**- additional return required to compensate for the impact of inflation on the real value of capital.

The Capital Asset Pricing Model also makes the assumption that investors should be compensated in two ways: time value of money and risk. The following shows how the three allowances are expressed and results in a rate of return for the investor.

$$\mathbf{ra = rf + \beta a(rm - rf)}$$

Where:

rf= Risk free rate (time value of money)

βa= Beta of the security

rm= Expected market return

3.4.3 Risk Free Rate

The risk free rate is properly defined as the time value of money and is represented by **rf**. **Rf** compensates the investors for placing money in any investment over a period of time. The amount of compensation will vary based on the length of time, the amount of the initial capital investment; the risk associated with the investment and external market influences. The latter part of the formula **(βa(rm- rf)** represents risk and calculates the amount of return needed for the investor to take on any additional risks. This is calculated by taking **β** (beta, which compares return of assets over a period of time) and compares it to the market premium **(Rm-rf).**

3.5 The Arbitrage Pricing Theory

The Arbitrage Pricing Theory was developed by Ross in 1976. This theory reviews the expected risk premium in relation to the expected market risk associated with the share. The act of arbitrage deals with taking profits for zero risk. In theory, this can occur when a portfolio's expected risk premium is = 0. The risk premium relates to the additional return required from an investor who holds a risky investment. The equation below illustrates risks associated with the each asset. The risks factors refer to market activity like interest rate increase or decreases, changes in the inflation rate, currency risks and a list of other market risks.

$$\mathbf{ERj=Rf+\beta\,1(ERfactor1-Rf)+\beta\,2(ERfactor2-Rf)+\ldots+uj}$$

- ERj= expected ROR on j
- ERfactor1=expected return on macro factor 1
- β1= the sensitivity of security j to factor 1
- uj= random deviation based on unique events affecting returns

The following bullet points are a list of assumptions associated with The Arbitrage Theory:

- ROR on share depends partly on macroeconomic factors and specific events in the company.
- Diversification can eliminate specific risk, leaving only market risk.
- Specifies the returns as a function of multiple macroeconomic factors that the market portfolio depends.

The limitations that are associated with the arbitrage theory are the fact that the theory only focuses on most influential factors that may affect returns (e.g. industrial production, inflation, personal consumption, money supply and interest rates). Although some specific risks are considered, they do not have as much importance as the market risk associated with the portfolio assets. Another limitation of the theory is that weighting have not been established for the market factors. The last limitation of this theory is that it sill in developmental stages and cannot be properly applied to diversifying one's portfolio.

4. Risk and Return Calculations

One question that can be asked in reference to portfolio diversification is whether or not portfolio risk can be minimized through portfolio diversification? After properly researching the topic, it has been determine that risk can be minimized through choosing various assets that have negative correlations in order to mitigate the risks associated with each individual asset. In dealing with risk, diversification can significantly reduce specific risk associated with a company but cannot entirely eliminate market risks that derive from macroeconomic factors.

In measuring risk, it is imperative to keep in mind that the degree of correlation and weight of each asset is key to properly calculating risk. The following equation is a standard deviation equation and can be used to measure different potential capital investments of a firm. In terms of A and B, 'A' can be denoted as Debt and 'B' as Equity. In this situation, a company can determine the degree or weight of risk involved in either choosing to issue debt or issue equity and also determine the potential rate of return on either investment.

$$\sigma p = \sqrt{\infty 2\ \sigma 2A + (1- \infty)2\ \sigma 2B + 2\infty(1-\infty)covAB}$$

- ∞= proportion of portfolio invested in A
- (1- ∞)= proportion of portfolio invested in B
- σA2= variance of return on A
- σB2= variance of return on B
- covAB= covariance of returns from A & B (measures the relations between A&B)

4.1 Covariance and Correlation in Measuring Risk

As defined earlier in the key terms, the correlation describes how the each asset behaves in relation to one another. The correlation of AB is significant because it will show the investor or financial manager to what degree each asset (A and B) will relate to each other. Below is the formula for the correlation of A and B returns.

Correlation coefficient between A& B returns = r_{AB}= cov_{AB}/ σ_A* σ_B

The covariance is a statistical measure of correlation of the fluctuations of two different quantities. In finance, covariance is applied to the annual rates of return of different investments, to measure the correlation of their year-to-year fluctuations in performance. Below illustrates the formula for the covariance for A and B:

Covariance, cov_{AB} between A & B = cov_{AB}= $\Sigma[p_i(R_A-E_{RA})(R_B-E_{RB})]$

Why do investors buy shares? Investors put capital into investments for the anticipated dividends and capital gains. In order to calculate the returns of for a shareholder, an equation can be applied.

Total Shareholder Return

t= any holding period

j= company

Rjt= % of return from shares

Djt= dividends per share

Pjt= share price

$$R_{jt}= D_{jt}+ \frac{(P_{jt}-(P_{jt-1})}{P_{jt-1}} * 100$$

For example, if the share price of Company X in March 2005 is 209p and decreases to 120p in March 2006, the actual returns from the shares owned can be calculated to reflect the following:

Share price at the end of March 2005= 209p
Share price at the end of March 2006= 120p
Net dividend paid during 2005-2006= 5p per share

$$5p + \frac{(120p - 209p) * 100}{209p} =$$

$$= \frac{-84p * 100}{209p} = -40.2\%$$

The above example illustrates how shareholders can lose a capital gain instead of earning dividends on the capital market. With any investment, there are still risk involved even after an investor employs risk mitigating tactics like diversification and using Theories and other financial experts and tools available.

5. Mean Variance Optimisation

Mean-variance theory was developed in the 50's and 60's by Markowitz, Tobin, Sharpe, and Lintner, among others. It is still called Modern Portfolio Theory (MPT) by some people. Mean-variance theory continues to be the main workhorse on which analytical portfolio management is based. The equilibrium version of mean-variance theory is called the Capital Asset Pricing Model (CAPM).

The goal of the theory is to optimally invest funds in wide variety of assets. It is a quantitative tool, allowing making investment decisions by considering the trade-off between risk and return. There are single and multi period mean variance optimisers. **Single-Period MVO** considers designing of portfolio for single upcoming period and maximising return considering presumed level of risk. **Multi-Period MVO** is a strategy, rebalancing portfolio at the end of each period.

5.1 Single Period Problem

- [11]Inputs:
  - The expected return for each asset
  - The standard deviation of each asset (a measure of risk)
  - The correlation matrix between these assets
- Output:
  - The efficient frontier, i.e. the set of portfolios with expected return greater than any other with the same or lesser risk, and lesser risk than any other with the same or greater return.

11 http://www.effisols.com/basics/MVO.htm

5.2 Multi-period Problem

- Input:
  - The full historical data set
- Desired output:
  - The Geometric Mean Frontier; i.e. the set of rebalanced portfolios with greater geometric mean return than any other with the same or lesser standard deviation, and lesser standard deviation than any other with the same or greater geometric mean return

5.3 Benefits of MV optimiser

- [12]Satisfaction of client objectives and constraints.
- It is possible to integrate client constraints and objectives in MVO, which enables individual approach to each client, dependant on his risk acceptance level.
- Control of portfolio risk exposure.
- It is possible to control risk exposure of portfolio with the help of MVO.
- Implementation of style objectives and market outlook.
- It is possible to reflect company's investment style, philosophy and market outlook in MVO.
- Efficient use of investment information.
- MVO is designed to reflect all the relevant information available for the optimisation of the portfolio.

[12] The Markowitz optimisation enigma

- Timely portfolio changes.

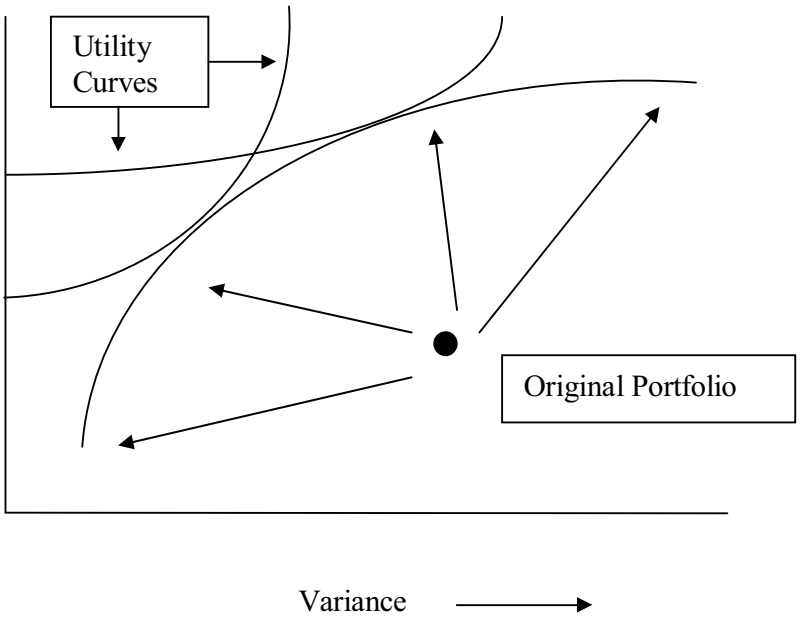
- MVO is capable of processing large volumes of data very quickly, which is useful for large organisation, that need to incorporate this information in there portfolios in order to reflect the changes in environment.

5.4 The Markowitz Optimization Enigma

The MVO is recognised as the foundation, for the most modern portfolio diversification models. But majority of investors avoid using it. What is the reason for it? It is difficult to use compared with informal traditional ways, to which managers are used to. Also there are political reasons for not using it. Using MVO suggests significant changes in the structure of organisation and to the investment management processes. More quantitative investment processes should be developed within the company. To use MVO Company needs thoroughly knowledge of basic statistical concepts and modern portfolio theories. Experienced investors have experimented with MVO. They abandoned the effort when found their portfolios unintuitive and without obvious investment value. Those listed might be the most obvious answers for Markowitz Optimization Enigma.

5.5 Markowitz Mean-Variance Efficient frontier

[13]Classical MVO assumes that investor chooses portfolio with maximum level of return for the level of risk that he is prepared to take on. Optimiser selects securities and proportion of funds to invest in chosen assets. "The set of optimal portfolios for all possible levels of portfolio risk defines the MV efficient frontier"



On the chart you can observe efficient frontier and utility curves (level of excepted risk) and on the tangent place of utility curve and efficient frontier lays the best options of the portfolio.

[13] The Markowitz optimisation enigma

5.6 Exact VS Approximate MV Optimizers

There are available two types of MV optimisers, Exact and Approximate. The differences are: processing time; entire frontier vs. single point solution; maximum size of the optimisation universe; and the ability to operate on standard personal computers.

Exact MVO assumes that it can find a solution for the entire efficient frontier. The drawbacks are relatively small universe size, long computing time and it dose not include transaction costs.

Approximate MVO is sufficient for large organisations and its computing abilities compared to latter is much faster. Limitation is that it calculates only one optimal portfolio near the efficient frontier as opposed to exact one.

6. Conclusion

It can be concluded that firstly, investors should use diversification to optimize their portfolios. Secondly, investors must keep in mind:

- The risk associated with each asset
- Correlation between assets within the portfolio and correctly weighing

Finally, risks can be minimized to a certain extent with the tool and theories mentioned in this report. But investors should be aware that some level of risk always remains.

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