Covariance

No consideration was given to the risk of an individual stock, what mattered was its covariance with other stocks and therefore its impact on the portfolio as a whole.

Covariance is a measure of the degree of the degree to which two assets move together relative to their individual mean values over time. A positive covariance means that the rates of return for two investments tend to move at the same direction according to their individual means during the same time period. While a negative covariance indicates that the rates of return of two assets tend to move in different directions relative to their means at a certain time period.

Covariance is affected by the variability of the two individual return series.

Correlation Coefficient - Diversification

Companies, by pooling their money enable future investors to hold fractional shares of many different securities. In fact, the exact definition of diversification is to spread a portfolio over many investments to avoid excessive exposure to any one source of risk. Portfolio diversification is important in today's market. Market swings create volatility. Having non-correlated investments in your portfolio can provide stability.Consequently, in order to control a portfolio risk investors should use the diversification method whereby investments are made in a large variety of assets for the purpose of reducing risk.

The risk of individual assets in a portfolio should be measured in the context of their effect of return on the overall portfolio variability. In order to illustrate the effect of diversification, we should take under consideration covariance and correlation theories. In fact, a way to measure covariance is to look at assets' returns or deviations from expected value.

When in two assets your correlation coefficient equals to 1, it is like you have one asset. This happens because the two assets will move to the same direction implying a high portfolio risk. In plain words, it is better to put a lot of different stocks in the same portfolio because the risk will be diminished. Consider about having two extremes; a portfolio with one stock, and a portfolio with 10 stocks. In the first case, if the stock goes down, you will lose a lot of money. In the second case, though, if some stock goes down, there will not be a great effect in your portfolio. In fact, the portfolio has the opportunity to increase its expected returns from its other stocks. Another benefit of the second case is the fact that the companies should come from different industries so that the one stock does not affect the others.

As a result, diversification implies that a portfolio should have stocks from different industries (negative correlation), while the more the stocks inside the portfolio, the less the risk inside that portfolio. The ideal case in a portfolio is when the correlation coefficient is close to -1. This figure will reduce the standard deviation of the portfolio; in other words, the risk of all the assets included in the portfolio will be diminished a lot.

A value of -1 indicates a perfect negative relationship between the two return series such that when one stock's rate of return is above its mean, the other stock's rate of return will be below its mean by the comparable amount. In other words if the assets have a low correlation level, their returns are moving in a opposite direction, which is very favorable for our portfolio.

Standard deviation

Stock prices of firms can be positively correlated where stock prices of different firms move in the same direction because they are affected by similar factors. Conversely they can be negatively correlated where they move in different directions because they are not affected by the same factors.
The latter means the correlation coefficient ρ is lower than zero and if such stocks are incorporated in a portfolio, it will tend to decrease the overall risk (standard deviation) of the portfolio dramatically.

The former means the correlation coefficient ρ is higher than zero and by adding a stock with a correlation to other stocks of less than 1, it will reduce the portfolio's risk since a portfolio's standard deviation is not equally weighted of individual stock's standard deviation as can be seen in the following formula:

$$\sqrt{\sigma^2} = \sum_{j=1}^{n} \sum_{i=1}^{n} w_j w_i Cov\ (r_j, r_i)$$

where $\sqrt{\sigma^2}$ is the standard deviation

w_i the weight in stock i

w_j is the weight in stock j

$Cov(r_j, r_i)$ is the covariance of stocks i and j

The standard deviation for a portfolio of assets encompasses not only the variances of the individual assets, but it also includes the covariances between all the pairs of the assets of the portfolio. In essence, this formula reduces to the sum of the weighted covariances as the number of assets rises. When more assets are added to a portfolio, there are two effects to the standard deviation; that is, the variance of returns and the different covariances of the assets. As the amount of assets increase in a portfolio, the different covariances are greater than the covariance of the asset that you want to add.

The results of the efficient portfolio analysis has been summarised in Exhibit 5. Further analyses of the results have been illustrated through Exhibit 6. Exhibit 5 clearly indicates that as the number of constituents of an efficient market portfolio increases the standard deviation, as measured by the minimum variance portfolio point in a Markowitz framework, reduces. These results suggest that the unique or diversifiable risk will be removed through the diversification of the portfolio. The smooth reduction in the standard deviation for all efficient portfolios starting from a standard deviation of 0.043 (for the efficient portfolio consisting of 5 securities) to a

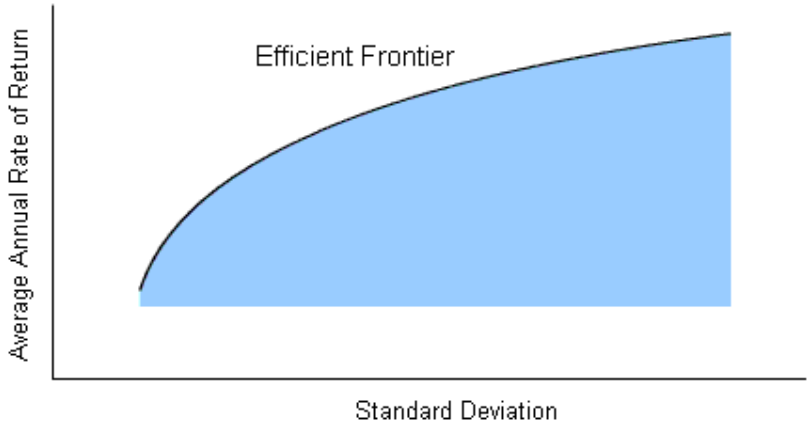
standard deviation of 0.034 (for the efficient portfolio consisting of all the securities) follows what theoretical literature predicts.

The final global minimum variance portfolio, which consists of all stocks, records a standard deviation of 0.034. This remaining risk is to be expected since the all well diversified portfolios include a remaining quantity of risk often referred to as the "market risk" or "unsystematic risk". The remaining market risk on the portfolio accrues from the impacts of macroeconomic variables on all individuals stocks held.

The results of the diversification process have been illustrated through Exhibit 5 - the increase in the number of constituents has effectively reduced the standard deviation of the portfolio and so reduced the burden of unique risk for the investor. Exhibit 6 shows a graphical representation of the same results and exhibit 7 shows the standard deviation and the number of companies.

An example of why stocks have lower correlation than 1 is that stocks are affected differently by economic news like changes in interest rates, exchange rates, oil prices, etc. For instance, the effect of high oil prices may be beneficial for the oil industry but harmful to the airline industry as we have seen in the recent past. So risk is not just simply the volatility of each stock but also a measure of how it reacts to other stocks in a portfolio

Efficient Frontier



A portfolio is said to be *efficient* if there is no portfolio having the same standard deviation with a greater expected return and there is no portfolio having the same return with a lesser standard deviation. The *efficient frontier* is the collection of all efficient portfolios.

It's clear that for any given value of standard deviation, you would like to choose a portfolio that gives you the greatest possible rate of return; so you always want a portfolio that lies up

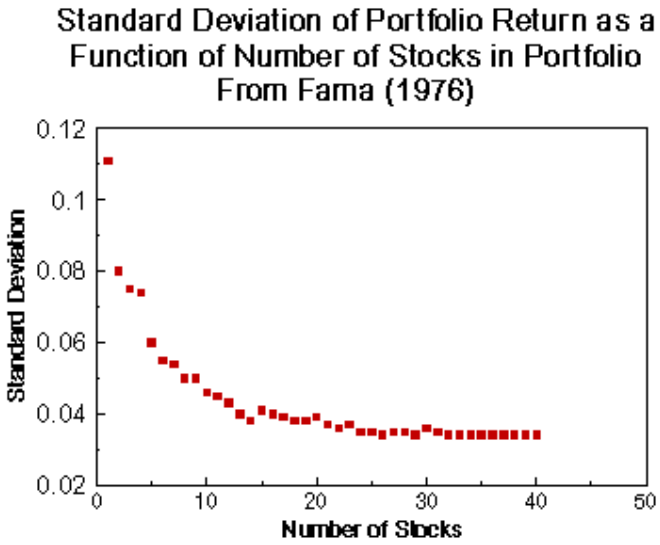
along the efficient frontier, rather than lower down, in the interior of the region. This is the first important property of the efficient frontier: it's where the best portfolios are.

The second important property of the efficient frontier is that it's *curved*, not straight. This is actually significant. In fact, it's the key to how diversification lets you improve your reward-to-risk ratio.

In statistical terms, this effect is due to lack of covariance. The smaller the covariance between the two securities, the more out of synchronization they are, the smaller the standard deviation of a portfolio that combines them. The ultimate would be to find assets with *negative* covariance (the best years of one happen during the worst years of the other, and vice versa).

III. More Securities and More Diversification

Now consider what will happen as you put more assets into the portfolio. You will find that you can reduce the standard deviation of the portfolio by mixing across several assets rather than just two. Each point represents an equally-weighted combination of assets; from a single stock to two, to three, to thirty, and more. Notice that, after 30 stocks, diversification is mostly achieved. There are enormous gains to diversification beyond one or two stocks.



(Courtesy Campbell Harvey)

If you vary the portfolio weights, rather than keeping them equal, the benefits are even greater; in fact, you have to calculate the standard deviation of every conceivable mixture of the securities. Nonetheless, you would find that there is a set of portfolios which provide the lowest level of risk for each level of return, and the highest level of return for each level of risk. By considering all combinations of assets, a special set of portfolios stand out.

For any level of risk, the efficient frontier identifies a point that is the highest returning portfolio in its risk class. By the same token, for any level of return, the frontier identifies the lowest risk portfolio in that return class. Notice that it extends from the maximum return portfolio (actually a single asset) to the minimum variance portfolio. The efficient frontier has a portfolio for everyone; actually, there are an

infinite number of points in the set, corresponding to the infinite variation in investor preferences for risk.