Question1

A company supplying parts to a large customer receives forecasts of the expected demand in advance of the delivery date. One forecast is received one month ahead of delivery and a revised forecast is received one week ahead of delivery (i.e. three weeks after the first forecast). Finally, the actual requirements are indicated electronically on the delivery date. The following table shows that the data sent to the supplier covering a period of six months (twenty four weeks).

Week #	Forecast Demand 1 Month Ahead	Forecast Demand 1 Week Ahead	Actual Demand
1	2450	2401	2321
2	2560	2520	2052
3	2730	2669	1680
4	2960	2850	2484
5	3180	2995	1380
6	3170	3085	2129
7	3170	3200	2221
8	3180	3295	2665
9	3240	3295	2052
10	3366	2332	2298
11	2675	1931	2054
12	1813	1412	1234
13	3365	2151	2409
14	2330	2259	1440
15	1953	1927	1980
16	2760	2700	2808
17	2760	2730	1980
18	2889	2812	1800
19	2980	3081	1908
20	3090	3101	1332
21	3180	3132	2148
22	3122	2907	1678
23	3305	3051	2412
24	2577	2371	2124

(i) Does the data indicate that the revised (one week) forecast is significantly more accurate than the first (one month) forecast?

In order to find out whether the revised forecast is significantly more accurate than the first forecast, we can use t-test to test whether the means are equal for two populations.

At first, we calculate the errors of both forecasts, which give us the following data:

Week	Error 1 month ahead	Error 1Week ahead	Difference between 2 errors
1	-129	-80	49

2	-508	-468	40
3	-1050	-989	61
4	-476	-366	110
5	-1800	-1615	185
6	-1041	-956	85
7	-949	-979	-30
8	-515	-630	-115
9	-1188	-1243	-55
10	-1068	-34	1034
11	-621	123	744
12	-579	-178	401
13	-956	258	1214
14	-890	-819	71
15	27	53	26
16	48	108	60
17	-780	-750	30
18	-1089	-1012	77
19	-1072	-1173	-101
20	-1758	-1769	-11
21	-1032	-984	48
22	-1444	-1229	215
23	-893	-639	254
24	-453	-247	206

In order to test whether the revised forecast is more accurate than the first one, the paired t-test could be used.

Define the question: is the result from the revised forecast more accurate than that of the first forecast?

Identify the appropriate model: as the mean difference is required and samples are small and not independent, we choose paired t-test.

Difference could be in either way; therefore, it is a two-tailed test.

Formulate the Null hypothesis and the Alternative hypothesis (H_0 and H_1):

H_0: $\mu_1 > \mu_2$

H_1: $\mu_1 \leq \mu_2$

The level of significance is $\alpha = 5\%$, then $\alpha/2 = 2.5\%$. With degrees of freedom, $v=23$, from Statistical Table, 5, $\boldsymbol{t_{crit}} = \mathbf{2.069}$.

$t_{calc} = \frac{\bar{d}-k}{\hat{\sigma}_{\bar{d}}/\sqrt{n}}$ = -2.77, with $\bar{d}$=191.58, $\hat{\sigma}_{\bar{d}}$=338.59

$t_{calc} > t_{crit}$

$\therefore$ Reject H_0. In other words, the result of revised forecast is more accurate than that of the first forecast.

t-Test: Paired Two Sample for Means

	Variable 1	Variable 2
Mean	-842.3333333	-650.75
Variance	227542.7536	323320.3
Observations	24	24
Pearson Correlation	0.804134873	
Hypothesized Mean Difference	0	
df	23	
t Stat	-2.771992028	
P(T<=t) one-tail	0.00542241	
t Critical one-tail	2.068657599	
P(T<=t) two-tail	0.010844821	
t Critical two-tail	2.397875057	

(ii) The forecasting method assumes that the errors are normally distributed. Does the data support this assumption for each of the two forecasts?

In this case, the "Goodness of Fit" χ^2-test can be applied to testify whether Normal Distribution model is suitable or not.
(1) The errors of the first (1 month ahead) forecast

Define the question: does a Normal distribution model fit the data?

Identify the appropriate model: it is obviously a "Goodness of Fit" question, so χ^2-test can be used.

As "fit" can be "too bad" to accept H_0, but it is can never be "too good" to accept H_0. Therefore, it is a One-tailed test.

Formulate the Null hypothesis and the Alternative hypothesis (H_0 and H_1):

H_0 : Normal distribution model does fit the data

H_1: Normal distribution model does not fit the data

The level of significance is set at α=5%.

The data are rearranged by chi-square test of Statpro package.

Frequency table and normal test for Error 1 month ahead				
Upper limit	Category	Frequency	Normal	Distance measure
-1800	<=(-1800)	1	0.536	0.401
-1500	(-1800)-(-1500)	1	1.480	0.155
-1200	(-1500)-(-1200)	1	3.425	1.717
-900	(-1200)-(-900)	9	5.405	2.391
-600	(-900)-(-600)	4	5.817	0.568
-300	(-600)-(-300)	5	4.270	0.125
	>(-300)	3	3.067	0.001
Total	7	24	24.000	5.359

Table 1. Excel StatPro package- Chi-square test for errors of one month ahead forecast

As displayed in the above table, the frequencies of the first three categories are blow 3. Because the χ^2 does not work very well with small frequencies, the categories of <=(-1800), (-1800)-(-1500), and (-1500)-(-1200) are combined to together, which gives the number of cell k=5.

The degrees of freedom ν equal to k-p-1, where k is the number of cells, p is the number of parameters estimated from the sample data of the proposed distribution. In the case of normal distribution, the value of the sample mean and standard deviation are required to calculate the Expected values, therefore, p=2. $\therefore$ $\nu=5-2-1=2$

From Statistical Table 7, with ν=2, χ^2_{crit}=5.991

From Table 1. the total distance measures equals to the Chi-square statistics 5.359, which is calculated by the Chi-square test in StatPro package in Excel. The Chi-square test :$\chi^2_{calc} = \sum \boldsymbol{(O-E)^2/E}$, which gives us χ^2_{calc}= 5.359.

Therefore, $\chi^2_{calc} < \chi^2_{crit}$, H_0 is accepted. As a result, the normal distribution is an adequate model for the data of errors of first forecast.

(2) The errors of second (one week ahead) forecast

Define the question: does a Normal distribution model fit the data?

Identify the appropriate model: it is obviously a "Goodness of Fit" question, so χ^2-test can be used.

As "fit" can be "too bad" to accept H_0, but it is can never be "too good" to accept H_0.

Therefore, it is a One-tailed test.

Formulate the Null hypothesis and the Alternative hypothesis (H_0 and H_1):

H_0 : Normal distribution model does fit the data

H_1: Normal distribution model does not fit the data

The level of significance is set at α=5%.

The data are rearranged by chi-square test of Statpro package.

Frequency table and normal test for Error 1Week ahead				
Upper limit	Category	Frequency	Normal	Distance measure
-1769	<=(-1769)	1	0.591	0.284
-1469	(-1769)-(-1469)	1	1.211	0.037
-1169	(-1469)-(-1169)	3	2.543	0.082
-869	(-1169)-(-869)	5	4.068	0.213
-569	(-869)-(-569)	4	4.959	0.185
-269	(-569)-(-269)	2	4.604	1.473
	>(-269)	8	6.024	0.648
Total		24	24.000	2.922

Table 1. Excel StatPro package- Chi-square test for errors of one week ahead forecast

As displayed in the above table, the frequencies in categories of <=(-1769) and (-1769)-(-1469) are blow 3. Because the χ^2 does not work very well with small frequencies, these two categories are combined to together into category (-1469)-(-1169), which gives the number of cell k=5.

The degrees of freedom ν equal to k-p-1, where k is the number of cells, p is the number of parameters estimated from the sample data of the proposed distribution. In the case of normal distribution, the value of the sample mean and standard deviation are required to calculate the Expected values, therefore, p=2. ∴ $\nu=5-2-1=2$

From Statistical Table 7, with ν=2, χ^2_{crit}=5.991

From Table 1. the total distance measures equals to the Chi-square statistics 2.992, which is calculated by the Chi-square test in StatPro package in Excel. The Chi-square test :$\chi^2_{calc} = \sum \boldsymbol{(O-E)^2/E}$, which gives us χ^2_{calc}= 2.992.

Therefore, $\chi^2_{calc} < \chi^2_{crit}$, H_0 is accepted. As a result, the normal distribution is an adequate model for the data of errors of weekly forecast.

(iii) Does it seem that the Actual Demand data are normally distributed?

The same method as previous section can be used to test whether the actual demand data are normally distributed.

Define the question: does a Normal distribution model fit the data?

Identify the appropriate model: it is obviously a "Goodness of Fit" question, so χ^2-test can be used.

As "fit" can be "too bad" to accept H_0, but it is can never be "too good" to accept H_0. Therefore, it is a One-tailed test.

Formulate the Null hypothesis and the Alternative hypothesis (H_0 and H_1):

H_0 : Normal distribution model does fit the data

H_1: Normal distribution model does not fit the data

The level of significance is set at α=5%.

The data of actual demand are rearranged by chi-square test of Statpro package.

Frequency table and normal test for Actual Demand				
Upper limit	Category	Frequency	Normal	Distance measure
1234	<=1234	1	0.674	0.158
1534	1234- 1534	3	2.158	0.328
1834	1534- 1834	3	4.912	0.744
2134	1834- 2134	8	6.759	0.228
2434	2134- 2434	6	5.626	0.025
	>2434	3	3.871	0.196
Total		24	24.000	1.679

Table 1. Excel StatPro package- Chi-square test for data of actual demand

As displayed in the above table, the frequency in categoriey of <=1234 is merely 1, which is blow 3. Because the χ^2 does not work very well with small frequencies, this category is combined with the below one 1234- 1534, which gives the number of cell k=5.

The degrees of freedom ν equal to k-p-1, where k is the number of cells, p is the number of parameters estimated from the sample data of the proposed distribution. In the case of normal distribution, the value of the sample mean and standard deviation are required to calculate the Expected values, therefore, p=2. $\therefore$ $\nu=5-2-1=2$

From Statistical Table 7, with ν=2, χ^2_{crit}=5.991

From Table 1. the total distance measures equals to the Chi-square statistics 1.679, which is calculated by the Chi-square test in StatPro package in Excel. The Chi-square test :$\chi^2_{calc} = \sum \boldsymbol{(O-E)^2/E}$, which gives us χ^2_{calc}= 1.679.

Therefore, $\chi^2_{calc} < \chi^2_{crit}$, H_0 is accepted. As a result, the normal distribution is an adequate model for the data of actual demand.

(iv) Comment on the answer to part (iii)

The result that the actual demand is normally distributed is expected by us. A normal distribution model can be described by two parameters: the mean and the variance. The data are normally distributed with the same fundamental shape and symmetry, no matter what the actual distribution parameters.

The "Goodness of fit" is test is an effective tool to identity normality, as well as some correlation tests.

(v) If the supplier aims at 95% service level, how much safety stock would they need to hold?

As assumed, the lead time is 4 weeks,

$\mu = \mu_1 + \mu_2 + \mu_3 + \mu_4 = 2025 * 4 = 8100$ $\quad \sigma = \sqrt{\sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2 + \sigma_4{}^2} = 828$

(Demand $-\mu$)/ σ=1.65, (as service level is 95%, using Statistical Table 5, t=1.65)

Therefore, Demand = 1.65*828+8100=9466

As Demand = safety stock +σ

$\therefore$ Safety stock = 9466-8100=1366

Actual Demand

	Column1
Mean	2024.542
Standard Error	84.49738
Median	2053
Mode	2052
Standard Deviation	413.951
Sample Variance	171355.4
Kurtosis	-0.37667
Skewness	-0.25112
Range	1574
Minimum	1234
Maximum	2808
Sum	48589

Count	24

(vi) Comment on the assumptions make in the calculation of safety stock.

We assume the population is normally distributed, but the variance is unknown. And the sample size (24) is relatively small. Therefore, we use t-test, rather than z-test.

The leading time has an import impact on the safety stock level. We assume it is 4 weeks in this case.

Question 2

(a) A review of road transport infrastructure suggests that the level of investment is in decline. The table below shows the level of investment with the investment index in year zero taken as 100.

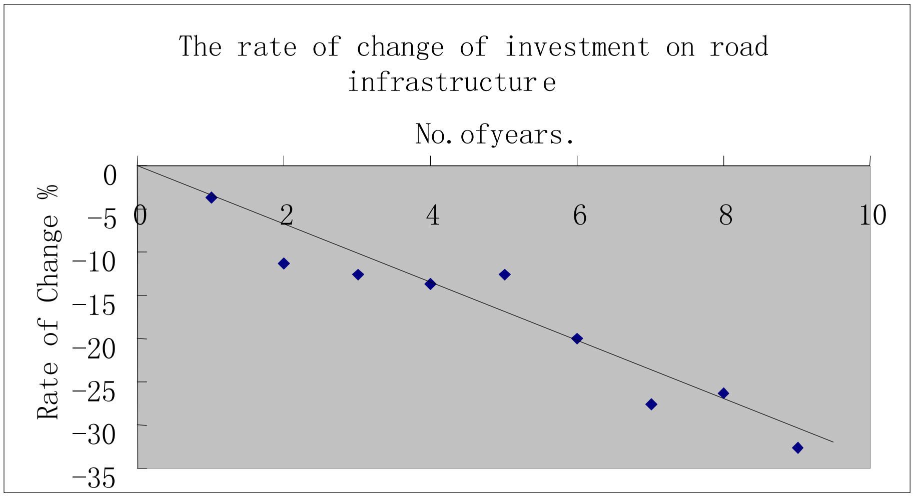
Year No.	Investment Index
0	100.00
1	96.25
2	88.75
3	87.50
4	86.25
5	87.50
6	80.00
7	72.50
8	73.75
9	67.50

(i) Plot a graph to show the rate of change

Firstly, the rate of change for each is calculated. We deduct the year 0's investment index from each sequent year's investment index, and divide the difference by 100, in order to get the percentage of rate of change.

Year No	Investment Index	Changes	Rate of decline
0	100.00	0.00	0.00%
1	96.25	-3.75	-3.75%
2	88.75	-11.25	-11.25%
3	87.50	-12.50	-12.50%
4	86.25	-13.75	-13.75%
5	87.50	-12.50	-12.50%
6	80.00	-20.00	-20.00%
7	72.50	-27.50	-27.50%
8	73.75	-26.25	-26.25%
9	67.50	-32.50	-32.50%

Using the tools in Excel, the following graph is created.



Graph 2.1, the rate of change of investment on road infrastructure

(ii) Does the data suggest that there is a significant relationship between investment and time, as suggested by the reviewer?

We assume that there is a significant relationship between investment and time, which is linear regression. As assumed, the equation is Y= a + bX, where x is time and y is investment.

Using Excel, the following data are transformed

X	Y	X^2	Y^2	XY
0	100.00	0	10000	0
1	96.25	1	9264.063	96.25
2	88.75	4	7876.563	177.5
3	87.50	9	7656.25	262.5
4	86.25	16	7439.063	345
5	87.50	25	7656.25	437.5
6	80.00	36	6400	480
7	72.50	49	5256.25	507.5
8	73.75	64	5439.063	590
9	67.50	81	4556.25	607.5
45	840	285	71543.75	3503.75

$$b=\frac{\sum xy-\frac{\sum x\sum y}{n}}{\sum x^2-\frac{(\sum x)^2}{n}}=\frac{3503.75-\frac{45\times 840}{10}}{285-\frac{45^2}{10}}=\mathbf{-3.348}$$

a =$\bar{y}$ - b$\bar{x}$ =840/10-(-3.348)×$\frac{45}{10}$ =99.066

Therefore, **the investment=99.066+ (-3.348)×number of years**.

An estimate of the error variance could be found out by the following calculations:

Source of Variance	Sum of Squares	Degrees of Freedom	Mean Square	Quantity estimated by Mean Square
Due to regression	$b^2\sum(x-\bar{x})^2$ $=b^2(\sum x^2 - \frac{(\sum x)^2}{n})=924.75$	1	M_1=924.75	$\sigma_0^2 + \sigma_R^2$
About regression(error)	By difference =59	n-2=8	M_0=7.375	σ_0^2
Total	$\sum y^2 - \frac{(\sum y)^2}{n}$ =983.75	n-1=9		

Therefore, the value of the error mean square $\hat{\sigma}_0^2$ = 7.375

And the standard deviation $\hat{\sigma}_0$ =2.716

In order to test the likelihood of any real relationship existing between the two variables, we choose F-test to be the assessment method.

H_0: σ_R^2 =0 or $\sigma_0^2 + \sigma_R^2$=0

H_1: $\sigma_R^2 > 0$ or $\sigma_0^2 + \sigma_R^2 > 0$

It is a one-tailed test, because a value of $\sigma_R^2 < 0$ does not have any sense.

The F value is calculated as: $F = \frac{M_1}{M_0} = \frac{924.75}{7.375} = 125.39$

At the level of significance is 5%, with v_G=1, v_L=8 (from the Degrees of Freedom column in above table), we find F_{crit} is 5.318, from Statistical Table 6(b).

As $F_{calc} > F_{crit}$, we reject H_0. Therefore, the conclusion is that there is a significant relationship between investment and times.

(iii) Produce an equation to show the rate of decline over the ten year period.

As is showed clearly in the previous graph, the rate of decline might follow the rules of linear regression, therefore, we assume the equation is Y= a + bX, where x is number of years and y is the rate of decline.

Using Excel, the following data are transformed

X	Y	X^2	Y^2	XY
0	0	0	0	0
1	-0.0375	1	0.001406	-0.0375
2	-0.1125	4	0.012656	-0.225
3	-0.125	9	0.015625	-0.375
4	-0.1375	16	0.018906	-0.55
5	-0.125	25	0.015625	-0.625
6	-0.2	36	0.04	-1.2
7	-0.275	49	0.075625	-1.925
8	-0.2625	64	0.068906	-2.1
9	-0.325	81	0.105625	-2.925
45	-1.6	285	0.354375	-9.9625

$$\mathbf{b}=\frac{\sum xy-\frac{\sum x\sum y}{n}}{\sum x^2-\frac{(\sum x)^2}{n}}=\frac{-9.9625-\frac{45\times(-1.6)}{10}}{285-\frac{45^2}{10}}=\mathbf{-0.033}$$

$$\mathbf{a=\bar{y}-b\bar{x}=(-1.6)/10-(-0.033)}\times\frac{45}{10}\ \mathbf{=-0.012}$$

Therefore, **the rate of decline=-0.012+ (-0.033)×number of years**.

(iv) Calculate the 95% confidence limits for rate of change

Furthermore, we can find out an estimate of the error variance, as shown below

Source of Variance	Sum of Squares	Degrees of Freedom	Mean Square	Quantity estimated by Mean Square

Due to regression	$b^2\sum(x-\bar{x})^2$ $=b^2(\sum x^2-\frac{(\sum x)^2}{n})=0.0898$	1	$M_1=0.0898$	$\sigma_0^{\ 2}$ + $\sigma_R^{\ 2}$
About regression(error)	By difference =0.0085	n-2=8	$M_0=0.0011$	$\sigma_0^{\ 2}$
Total	$\sum y^2-\frac{(\sum y)^2}{n}$ =0.0983	n-1=9		

Therefore, the value of the error mean square $\hat{\sigma}_0^{\,2}$ = 0.0011

And the standard deviation $\hat{\sigma}_0$ =0.033

The standard error of b is given by

$$\text{S.E.(b)}=\frac{\hat{\sigma}_0}{\sqrt{\sum(x-\bar{x})^2}}=\sqrt{\frac{\hat{\sigma}_0^{\,2}}{\sum(x-\bar{x})^2}}=\sqrt{\frac{0.0011}{82.5}}=0.0037$$

The 95% confidence limits gives us α=5%,α/2=2.5%, v=8, from Statistical Table 5,

Therefore, $\beta=b\pm t_{\alpha/2,v}\times$ S.E.(b)

$$=-0.033\pm2.306\times0.0037$$

$$\beta = -0.042 \text{ to } -0.024$$

(b)
A chemical filtration process was investigated to find the effect of different filter media on the speed of processing. Samples were collected from "Fast" filters and from "Slow" filters and the amount of a certain chemical ingredient was measured. The results are shown below. Does it seem that the amount of the chemical is significantly different between the two types of filter?

Explain your assumptions and your choice of tests.

"Fast" Filters	**"Slow" Filters**
8.4%	9.0%
9.8%	10.2%
12.2%	9.6%
12.6%	4.4%
13.0%	7.0%
9.2%	9.2%
13.6%	

In order to find out an effective method to test whether the amount of the chemical is significantly different between the two types of filter, t-test can be adapted. However, how to choose a specific t-test depends on the result whether the variances of two types of filters are equal or not by using F-test.

F-test:
Define the question: is there a difference in **variance** between the outputs of the two types of filters?

Identify the appropriate model: comparison of two variances $\therefore$ we use F-test

Because difference could be in either direction, $\therefore$ we use two-tailed test.

Formulate the Null hypothesis and the Alternative hypothesis (H_0 and H_1)

H_0: $\sigma_1^2 = \sigma_2^2$

H_1: $\sigma_1^2 \neq \sigma_2^2$

We set the level of significance α=5%, therefore, $\alpha/2$ =2.5%. $\upsilon_G = n_G - 1 = 6-1=5$,

$\upsilon_L = n_L - 1 = 7-1 = 6$. From Statistical Table 6(c), F_{crit} =5.988

Because $\hat{\sigma}^2 = \sum(X - \overline{X})^2 / n$

$$\therefore \hat{\sigma}^2{}_1 = \sum(x_A - \bar{x}_A)^2 \,/\, n \quad ; \quad \hat{\sigma}^2{}_2 = \sum(x_B - \bar{x}_B)^2 \,/\, n$$

Then we calculate the $\overline{X}_A$ and $\overline{X}_B$.

Because $\overline{X} = \sum x / n$,

$\therefore \overline{X}_A = \sum x_A / n = 78.8\% / 7 = 0.1126$

$\overline{X}_B = \sum x_B / n = 49.4\% / 7 = 0.0823$

Therefore,

$\hat{\sigma}^2{}_1 = \sum(x_A - \bar{x}_A)^2 / n = 0.000429$

$\hat{\sigma}^2{}_2 = \sum(x_B - \bar{x}_B)^2 / n = 0.000469$

As the variance of slow filter is greater than that of fast filter, we make $\hat{\sigma}^2{}_2$ be the numerator, and the $\hat{\sigma}^2{}_1$ be the denominator.

$\therefore$ $F_{calc} = \hat{\sigma}^2{}_2 / \hat{\sigma}^2{}_1 = 0.000469/0.000429 = 1.09$

$\therefore$ $F_{calc} < F_{critc}$

$\therefore$ Accept H_0

Therefore, there is no evidence of a significant difference in variance, which leads us to use the conventional t-test for difference between mean values of the two types of filters.

The following table contains useful data from Excel Data Analysis:

F-Test Two-Sample for Variances

	Variable 1	Variable 2
Mean	0.082333333	0.112571429
Variance	0.000469467	0.000428952
Observations	6	7
df	5	6
F	1.094449378	
P(F<=f) one-tail	0.449342319	
F Critical one-tail	5.987565126	

T-test:

Define the question: is there a significant difference in the **mean** output between the two filters?

Identify the appropriate model: because the population variance is not known and the mean is required, we use t-test.

As difference in either direction is possible, we choose to use two-tailed test.

Formulate the Null hypothesis and the Alternative hypothesis (H_0 and H_1)

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

We set the level of significance α=5%, $\alpha/2$=2.5%, as it is a two-tailed test.

From Statistical Table 5, with $v = 6 + 5 = 11$, t_{crit}=2.201

As calculated previously, $\overline{X}_A$=0.1126, $\overline{X}_B$=0.0823, and pooled standard deviation of

sample equals to $\sqrt{\frac{(nA-1)\hat{\sigma}^2{}_1+(nB-1)\hat{\sigma}^2{}_2}{nA+nB-2}} = \sqrt{0.0004473} = 0.0211$

$$t_{calc} = \frac{(\overline{X}_A - \overline{X}_B) - k}{\hat{\sigma}\sqrt{\frac{1}{n_A}+\frac{1}{n_B}}}$$

$$t_{calc} = \frac{(0.1126-0.0823)-0}{0.0211(\sqrt{\frac{1}{7}+\frac{1}{6}})} = 2.593$$

Therefore, $t_{calc} > t_{crit}$ $\therefore$ Reject H_0

There is significant difference between the outputs of the two filters.

The following table of data is abstracted from Excel Data Analysis.

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	Variable 2
Mean	0.112571429	0.082333333
Variance	0.000428952	0.000469467
Observations	7	6
Pooled Variance	0.000447368	
Hypothesized Mean Difference	0	
df	11	
t Stat	2.569655824	
P(T<=t) one-tail	0.013033185	
t Critical one-tail	2.200985159	
P(T<=t) two-tail	0.02606637	
t Critical two-tail	2.593092681	

Question 3

(a)

A study was conducted on the utilisation of machines in a press shop to determine the Actual Capacity. The approach used was "Activity Sampling". At random intervals of time the press shop was visited and the number of machines working was noted. The data are shown in the table below:

Number of machines working	
22	30
20	22
16	23
26	28
24	32
19	20
27	26
21	19
20	20
31	27
22	24
27	16
29	27
25	22
28	21

If the total number of machines was thirty-four,

(i) What was the average utilization of the press shop machines?

We set X is the number of machines working each time of visiting, therefore the mean of the sample is $\sum X$ / n, where n is the times of visiting.

$$\therefore \bar{X} = 714/30 = 23.8$$

Therefore, the utilization of the press shop machines is 23.8/34=70%

(ii) Suggest a model to represent the activity of the machines.

The Poisson distribution would be one of the suggested models, because the data representing the activity of the machines have the significant characteristics of a Poisson model. The number of times machine working is known, however, the number of visiting trials is unknown and could be large, with a constant measuring interval existing. In order to testify the suggestion, we use the "Goodness to fit" test to prove whether the Poisson distribution model fits or not.

Define the question: does a Poisson distribution model fit the data?

Identify the appropriate model: it is obviously a "Goodness of Fit" question, so χ^2-test can be used.

As "fit" can be "too bad" to accept H_0, but it is can never be "too good" to accept H_0. Therefore, it is a One-tailed test.

Formulate the Null hypothesis and the Alternative hypothesis (H_0 and H_1):

H_0 : Poisson distribution model does fit the data

H_1: Poisson distribution model does not fit the data

The level of significance is set at α=5%.

The data are rearranged by chi-square test of Statpro package.

Frequency table and normal test for Number of machine working				
Upper limit	Category	Frequency	Normal	Distance measure
16	<=16	2	1.009	0.975
19	16- 19	2	2.893	0.275
22	19- 22	10	6.191	2.344
25	22- 25	4	8.234	2.177
28	25- 28	8	6.808	0.209
	>28	4	4.866	0.154
Total		30	30.000	6.134

Table 3.1 Excel StatPro package- Chi-square test for numbers of machine working

As displayed in the above table, the frequencies of the first two categories are blow 3. Because the χ^2 does not work very well with small frequencies, the categories of <=16, and 16-19 are combined to together, which gives the number of cell k=5.

The degrees of freedom ν equal to k-p-1, where k is the number of cells, p is the number of parameters estimated from the sample data of the proposed distribution. In the case of Poisson distribution, the value of the sample mean is required to calculate the Expected values, therefore, p=1. $\therefore$ $\nu=5-1-1=3$

From Statistical Table 7, with ν=3, χ^2_{crit}=7.815

From Table 3.1, the total distance measures equals to the Chi-square statistics 6.134, which is calculated by the Chi-square test in StatPro package in Excel. The Chi-square test :$\chi^2_{calc} = \sum \boldsymbol{(O - E)^2 / E}$, which gives us χ^2_{calc}= 6.134.

Therefore, $\chi^2_{calc} < \chi^2_{crit}$, H_0 is accepted. As a result, the Poisson distribution is an adequate model for the data of machine working.

(iii) From the model, what is the lowest number of machines likely to be working at any one time?

(iv) What is the maximum number likely to be working?

(b)

Complaints were made about the level of pollutants in the discharge from a certain factory. The factory refuted the complaints by showing the results of their own analysis of the discharges. However, the Environmental Health Agency claimed the method of analysis used by the firm was faulty. A comparison was made over nine days using two methods of analysis in parallel to check the pollution levels. The results (in ppm.) are shown below:

Day #	Method A (firm's method)	Method B (EHA's method)
1	4	18
2	37	37
3	35	38
4	43	36
5	34	47
6	36	48
7	48	57
8	33	28
9	33	42

Does the data suggest the firm's method does underestimate the level of pollution?

<u>**Solution**</u>:

In order to test whether the firm's method underestimates the level of pollution or not, t-test could be used. If there is no difference between the variances of two populations, we will use Student's T-test; otherwise Aspin-Welch t-test will be used. Therefore, we need to do the F-test at first.

F-test:

Define the question: is there a difference in **variance** between the outputs of the two methods of analyzing the discharges?

Identify the appropriate model: comparison of two variances $\therefore$ we use F-test

Because difference could be in either direction, $\therefore$ we use two-tailed test.

Formulate the Null hypothesis and the Alternative hypothesis (H_0 and H_1)

$$H_0: \ \sigma_1^2 = \sigma_2^2$$

$$H_1: \ \sigma_1^2 \neq \sigma_2^2$$

We set the level of significance α=5%, therefore, $\alpha/2$ =2.5%. $\upsilon_G = n_G-1 = 9-1=8$, $\upsilon_L=n_L-1=9-1=8$. From Statistical Table 6(c), $\boldsymbol{F}_{crit}$ =4.433

Because $\hat{\sigma}^2 = \sum(X - \overline{X})^2 / n$

$$\therefore \hat{\sigma}^2{}_1 = \sum(x_A - \bar{x}_A)^2 \ / \ n \quad ; \quad \hat{\sigma}^2{}_2 = \sum(x_B - \bar{x}_B)^2 \ / \ n$$

Then we calculate the $\overline{X}_A$ and $\overline{X}_B$.

Because $\overline{X} = \sum x / n$,

$\therefore \overline{X}_A = \sum x_A / n = 303/9=33.67$

$\overline{X}_B = \sum x_B / n = 351/9=39$

Therefore,

$\hat{\sigma}^2{}_1 = \sum(x_A - \bar{x}_A)^2 \ / \ n$ =149

$\hat{\sigma}^2{}_2 = \sum(x_B - \bar{x}_B)^2 \ / \ n$=131.75

$\therefore \ F_{calc} = \hat{\sigma}^2{}_1 / \hat{\sigma}^2{}_2 = 149/131.75 = 1.13$

$\therefore \ F_{calc} < F_{critc}$

$\therefore$ Accept H_0

Therefore, there is no evidence of a significant difference in variance, which leads us to use the conventional t-test for difference between mean values.

The following table contains useful data from Excel Data Analysis:

F-Test Two-Sample for Variances

	Variable 1	Variable 2
Mean	33.66667	39
Variance	149	131.75
Observations	9	9
df	8	8
F	1.13093	
P(F<=f) one-tail	0.43305	
F Critical one-tail	4.43326	

T-test:
Define the question: is the **mean** of outputs using firm's method lower than that of the EHA method?

Identify the appropriate model: because the population variance is not known and the mean is required, we use t-test.

As "underestimate" is a defined direction, we choose to use one-tailed test

Formulate the Null hypothesis and the Alternative hypothesis (H_0 and H_1)

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 < \mu_2$$

We set the level of significance α=5%, as it is a one-tailed test.

From Statistical Table 5, with $v = 8 + 8 = 16$, t_{crit}=1.746

As calculated previously, $\overline{X}_A$=33.67, $\overline{X}_B$=39, and pooled standard deviation of sample equals to $\sqrt{\frac{(nA-1)\hat{\sigma}^2{}_1+(nB-1)\hat{\sigma}^2{}_2}{nA+nB-2}} = \sqrt{140.375}$=11.84

$$t_{calc} = \frac{(\overline{X}_A - \overline{X}_B) - k}{\hat{\sigma}\sqrt{\frac{1}{n_A}+\frac{1}{n_B}}}$$

$$t_{calc} = \frac{(33.67-39)-0}{11.85\left(\sqrt{\frac{1}{9}}+\sqrt{\frac{1}{9}}\right)} = -0.95$$

Therefore, $t_{calc} < t_{crit}$ $\therefore$ Accept H_0

There is no difference between the outputs of the two methods, and no evidence shows that the firm's method underestimates the level of pollution.

The following table of data is abstracted from Excel Data Analysis.

t-Test: Two-Sample Assuming Equal Variances

	Variable 1	Variable 2
Mean	33.66666667	39
Variance	149	131.75
Observations	9	9
Pooled Variance	140.375	
Hypothesized Mean Difference	0	
df	16	
t Stat	-0.954904852	
P(T<=t) one-tail	0.176915808	
t Critical one-tail	1.745883669	
P(T<=t) two-tail	0.353831617	
t Critical two-tail	2.119905285	