

Portfolio Assignment Mathematical Modelling SL Type II

In 1990 a firm of management consultants made a prognosis for the number of passengers traveling with Hurtigruten Bergen – Kirkenes. The prognosis is given by

$$f(x) = \frac{540000}{1 + e^{-0.15x}}$$

$x = 0$ corresponds to the year 1990, $f(0)$ is the number of passengers this year, $f(1)$ is the number of passengers traveling in 1991 etc.

Part 1

Task 1

How many traveled in 1990?

Using the given formula and the fact that $e^0 = 1$ I can rewrite the formula:

$$f(0) = \frac{540000}{1 + e^{-0.15 \cdot 0}} \rightarrow \frac{540000}{2} = 270000$$

According to the function, the number of passengers traveling in 1990 will be 270000.

Task 2

The prognosis tells us that one year 450000 passengers will travel Bergen – Kirkenes. Which year?

To answer this question, I will put $f(x) = 450000$ and then solve for “x” to figure out when the number of passengers will exceed 450000.

$$450000 = \frac{540000}{1 + e^{-0.15x}}$$

$$1 + e^{-0.15x} = \frac{540000}{450000}$$

$$1 + e^{-0.15x} = 1,2$$

$$e^{-0.15x} = 0,2$$

$$\dots | e^{-0.15x} = \dots | \dots |$$

$$-0,15x = \ln 0,2$$

$$\frac{\ln 0,2}{0,15} = 10,73$$

Hence, the number of passengers traveling will exceed 450000 in 2001, after 11 years.

Task 3

Show that

$$f'(x) = \frac{a}{(1 + e^{-0,15x})^2}$$

and find "a". Explain why "f" is an increasing function.

By using the quotient rule, I will find f'(x):

$$\left(\frac{a}{v}\right)' = \frac{a'v - v'a}{v^2}$$

$$a = 540000$$

$$v = 1 + e^{-0,15x}$$

$$f'(x) = \left(\frac{a}{v}\right)' = \frac{0 \cdot 1 + e^{-0,15x} - a \cdot e^{-0,15x} \cdot (-0,15)}{(1 + e^{-0,15x})^2}$$

$$\left(\frac{a}{v}\right)' = \frac{81000}{(1 + e^{-0,15x})^2} e^{-0,15x}$$

This tells us that "a" = 81000.

"x" represents the number of years. That means that "x" will always be a positive, since a negative amount of years is impossible. f'(x) will always be positive, given that "x" will always be a positive, and the fact that "e" is also a positive. That means, both numerator and denominator will always be a positive. Hence, the derivative will be positive, too. Furthermore, the function f(x) or "f" is an increasing function, since the value of the numerator is increasing "faster" than the value of the denominator.

Task 4

As time passes by, this prognosis (function) tells us that the number of passengers will tend to a certain value. Compute this value.

To find the value I will put in different values for "x" in the formula

$$f(x) = \frac{540000}{1 + e^{-0,15x}}$$

X	f(x)
100	540000
200	540000
500	540000

Task 4 continued:

There is another method to compute to what certain value the number of passengers will tend to. By using the formula

$$\lim_{x \rightarrow \infty} f(x) = \frac{540000}{1 + e^{-0.15x}}$$

it is possible to see that $e^{-0.15x}$ will tend to zero. That means that the denominator will tend to 1. Hence, $f(x)$ will tend to be 540000.

Task 5

Sketch the graph of "f".

Task 6

Compute $\int_0^7 f(x) dx$

For this calculation we just put in the values obtained throughout the portfolio:

$$\int_0^7 f(x) dx = \int_0^7 \frac{540000}{1 + e^{-0.15x}} = 2364881$$

As one can see, this is the calculation of the area under the graph from $x = 0$ to $x = 7$. This gives us the number of passengers that will travel with Hurtigruten Bergen – Kirkenes from 1990 – 1997.

Part 2

Graphic display

Task 7

Comment on the estimates obtained from the model given by the function “f” and the usefulness of this model.

If one plots the function against the provided point series, it is quite obvious that the predictions are mainly incorrect. However, to some extent it does match the actual figures. The model is or might be useful for the prediction of the number of passengers in the future. However, there is no correlation when predicting recession or time periods with a lower increase, as one can see when considering e.g. the period from 1992 to 1993.

Task 8

Use the display above (or other information) to find a model (function) for the development of the number of passengers on Hurtigruten Bergen – Kirkenes. Comment on your model.

If one plots the values given in the *graphic display* showing the actual number of passengers into the “STAT” function of a GDC, one finds the exponential function

$$f(x) = 254865 \cdot e^{0,0615x}$$

as the most accurate function.

With this function it is possible to observe predictions of the number of passengers to a specific degree of certainty which is about 90%. This exponential function is a good alternative to the given one.

It is probably possible to produce a model that is way more accurate than the provided one in this assignment (*graphic display*). Nevertheless, there is no function that could possibly match the number of passengers exactly of Hurtigruten Bergen – Kirkenes. The only to find out is to wait and see. The predictions in this portfolio are just a possible approach to what the firm of management consultants expect and forecast.